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THE MATHEMATICS TEACHER

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THE MATHEMATICS TEACHER

Volume XXXII Number 7

Edited by William David Reeve

The Mathematics of Most Value to Pupils of Science

By Ira C. Davis
University of Wisconsin, Madison, Wisconsin

All divisions of subject matter have some responsibilities in teaching pupils how to use mathematics. The pupil's ability to use and understand the principles of mathematics appears to depend on the number and variety of applications made. Experienced science teachers realize they have many splendid opportunities to give pupils new experiences in using and applying mathematics. But how far can science teachers go with these new experiences and problems? Two questions must be raised at the outset—first, what mathematics do the pupils really understand; and second, what additional mathematics will need to be taught if the pupils are to comprehend the new experiences? This is where the trouble begins. How is the science teacher to know what mathematics his pupils understand, and does the ordinary science teacher know how to teach the additional mathematics they need to learn? Possibly it will help to answer these two questions by asking another—how can the science teacher help pupils apply the mathematics they are supposed to have learned?

The following comments and suggestions are made on the assumption that the teaching of both science and mathematics can be improved by having teachers of these subjects get together to study the problems involved. What I have to say refers to the things science teachers would like the mathematics teachers to do. The science teachers would also like to hear what the mathematics teachers have to suggest.

I might say in the beginning that science teachers do not use any mathematics that has not been taught in mathematics classes. However, science teachers soon discover that many of their pupils do not know how to use mathematics in solving scientific problems. It does not help to say that the mathematics teachers are at fault. Neither does it help to omit all science problems involving the use of mathematics. As I look at it, that is the wrong thing to do, for it means avoiding a difficulty rather than facing it. So many of our teachers now make the excuse for not teaching by saying their pupils cannot learn anything. What a feeble excuse that is! Would it not be much better to say—pupils can learn mathematics if we discover how it should be taught? After all, isn't that the most interesting and worthwhile problem we have in teaching?

What are some of the mathematical difficulties pupils have as observed by science teachers? First of all, pupils seem to be confused over the multiplicity of mathematical operations they perform. They are

surprised when told all they do is add, subtract, multiply, and divide, if squares and square root are omitted. Pupils know how to add, subtract, multiply, and divide fairly well, but the trouble is—they do not know *when to do it.* Is it not possible to simplify the presentation of arithmetic so pupils will know that all they do is perform the four fundamental operations (if square root is omitted)? All they can add in arithmetic is numbers, or parts of numbers. The science teachers wish the mathematics teachers would take more time to give meaning to these fundamental operations and less time to solving pages of problems for the mere sake of getting an answer.

Science teachers use the decimal system in practically all of the measurements and mathematical calculations they make. I do not see any real reason why pupils should continue to add fractions by the common denominator method. Why not change the fractions to decimals and then add the decimals? It is much easier, and I believe with a little practice it could be done more rapidly. All the pupils would need to do is learn the decimal equivalents of a few common fractions. How much simpler subtracting, multiplying, and dividing would become if decimals were used.

Are these fair questions for mathematics teachers to answer? Doesn't your emphasis on teaching fractions tend to perpetuate a system of measurement which is becoming obsolete? And if it isn't becoming obsolete, shouldn't it? How much time would be saved in arithmetic if the metric system of measurement were substituted for our English system?

Another question in connection with decimals—Isn't it possible to teach pupils where to place the decimal point when they multiply and divide, especially divide? Why is so much emphasis placed on the numbers to the right of the decimal point when they really have nothing to do with the placing of the decimal point? Why not teach pupils to glance at the problem and see how many numbers will precede the decimal point? Then, again, *too many pupils fail to realize the importance of placing the decimal point in the right position.* They seem to be satisfied if they get one somewhere. It may not be a serious matter to a pupil in a mathematics class if he writes an answer as .6 when it should be 6. It does make a great deal of difference in a science class if a table is said to be .6 of a foot in length when it really is 6 feet.

Science teachers wish that the mathematics teachers would stop making so many problems come out even. There may be some reason for doing this in the lower grades, but I see no reason for continuing to do the same thing in the junior and senior high schools. One pupil in physics class this year had these measurements for a table top—1.82 meters in length and 1.26 meters in width. When asked to calculate the surface area of the table top, he wanted to know whether he couldn't drop the decimals. Then I asked him what would happen to the dimensions of the table if he should drop them. He replied he didn't think it would make much difference. He was then asked to make the calculations both ways, and to his surprise he found that the area was only one square meter if he dropped the decimals, and if he kept them the area was 2.29 square meters. He finally was convinced that the area of the table top was 2.29 square meters and not one square meter and that dropping the decimals did make some difference.

We seldom get answers in whole or even numbers when we make measurements in the laboratory. It happens so rarely that if we do get an answer in whole numbers, we suspect that we have made a mistake. We like to believe that we measure quite accurately in the laboratory. We would like to believe that we can get pupils interested in using exact measurements in their calculations. But it does not help us if mathematics teachers always give problems that come out even and, if they do not come out even, just drop the decimals

to make the problems easier. I sometimes wonder whether pupils do not know how to use decimals or whether they have been taught to be mathematically lazy.

Science teachers do not like to see less emphasis given to the teaching of squares and square root. The law of inverse squares is exceedingly important in science. Intensity of illumination, loudness of sounds, loss of heat by radiation, rate of vibration of pendulums, resistance of different sized wires, effect of tension on vibration rate of wires, and magnification in lenses, microscopes, and telescopes are just a few of the fundamental principles we try to teach which depend on the formula πr^2 for their complete understanding. This same formula is causing the difficulty in perfecting television. I hope some mathematics or science teacher will discover a method of teaching pupils how to use the formula πr^2 and really understand it. In our state testing program we find that more than 90 per cent of the pupils in physics classes fail to solve problems involving the formula πr^2.

Many other fundamental principles in science depend on the understanding of squares and square root. The formula for kinetic energy is $mv^2/2g$; for acceleration, $s = \frac{1}{2}at^2$; and for centrifugal force, $F = mv^2/$ gr. These formulas taken together form the foundation principles for machine design and for the safe driving of automobiles. It takes nine times the distance to stop a car going sixty miles an hour than to stop the same car going twenty miles an hour, neglecting the reaction time of the driver.

How many people understand why an automobile goes many more miles on a gallon of gasoline at lower speeds? Again, the law of inverse squares applies because the resistance of the air increases as the square of the speed. For water it is the cube of the speed. Why do airplanes go faster than automobiles, automobiles faster than boats, and why do we have streamlining in modern cars and planes? How many people can go to stores and judge the quantity of liquid they are getting by observing the diameter of the bottle?

Have I given enough illustrations to show you how important it is to have pupils really understand and know how to solve problems involving squares and square root? I am not convinced that the teaching of square root should be discontinued because some pupils have difficulty in learning it. Neither should it be omitted because some people through ignorance say it has no practical value. It has as much practical value as anything you teach in mathematics. Pupils in science must really understand the law of inverse squares and square root if they are to acquire mastery of several fundamental principles.

Science teachers wish that their pupils had a better understanding of units, how they are derived and used. What are the units of length, area, volume, force, weight, velocity, time, work, power, and many others that might be mentioned? Possibly a laboratory is needed to help make these units more meaningful. This involves, too, a rather accurate definition of units. Might I use the word *space* as an illustration? If I say the space between two windows is three feet, space means linear measure or length. If the floor space of a room is calculated, space means area. If I should say all matter occupies space, then space would mean volume. Can you imagine the confusion that exists in pupils' minds when they first hear the expression, "Air occupies space"? Up to this time they have not used the word space to mean volume. The saddest part of it all is that many science teachers never tell them it means volume. Again, what confusion there must be in the minds of boys and girls in the first grade when they see this statement in their science readers, "Space is vast." How can a pupil understand geometry unless he understands the word space? Isn't the increased difficulty in solid geometry due to the changed meaning of the word space?

Science teachers would like to have pu-

pils trained to begin with units of length, then expand them to units of area, then to units of volume, and on to units of force, weight, work, and power. Then too we wish pupils could define and illustrate the following terms, ratio, proportion, equation, simultaneous equations, quadratics, percentage, formulas, and graphs. When I say illustrate, I mean they should be able to create problems of their own and solve them according to the definitions given for each.

I wish I had kept a record of the number of pupils in advanced algebra whom I have asked to define a quadratic. So far I have not found one pupil who could define the term satisfactorily. Pupils can solve a quadratic equation, but they cannot tell you what it means or what it is. Very seldom do you find pupils who really understand simultaneous equations. They get a great deal of pleasure in solving the equations. But how many can create problems which may be solved by means of these equations? Seldom do they realize what the word simultaneous means. Rarely does it occur to them that the unknown factors must have the same values in the equations used for one problem. It seems to me the purpose in giving simultaneous equations is to teach pupils how certain types of problems may be solved by their use and not as training in mental gymnastics.

A boy came to me a few days ago very much elated over the fact he had solved two pages of problems in quadratic equations. When asked to define a quadratic, he was unable to do so. Neither could he tell what method he had used to solve the equations. He had copied the procedure given at the top of the page in the textbook. Little did he realize he had completed the square in each equation. When told he had done so, he could see no reason for doing it. When I multiplied each of the equations by four or nine, he was unable to solve them. With some help, he finally solved the new equations but he could not understand why the answers were the same.

I mention this incident to emphasize the differences in types of problems used in science as compared with mathematics. In mathematics many teachers give a more or less formal type of presentation for solving certain types of problems. Then they give many problems to illustrate the procedure they use! In science all kinds of problems may arise from one situation. We cannot limit the types of problems. We must take them as they come. We may use many different types of mathematical operations each day. This means that pupils must bring together the mathematical knowledge they have acquired and select that which is needed for each particular problem. In science pupils must make mathematics fit the problem, not the problems to fit the mathematics. This means that pupils must learn how to think mathematically if they are to succeed in solving the many problems which arise in science, especially the physical sciences.

I hope I have made it clear that science teachers want their pupils trained to think mathematically. They want their pupils to understand and know why they use a certain mathematical operation to solve many different types of problems. We think you should devote more time to real problems and less time to pure mental processes. Is it not possible in arithmetic, algebra, and geometry to give a wide variety of problems at regular intervals? Each group of problems would require for their solution practically all of the mathematical knowledge pupils had gained. With such a procedure, learning mathematics would become a continuous and growing process.

In conclusion might I say that the science teachers want the mathematics teachers to know they use mathematics in their measurements, in their calculations, and in their ways of thinking. We want you to know, too, we must use more and more mathematics to solve our increasingly complex problems. But we do not ask you to solve our problems for us. We expect to teach mathematics with you and

to help pupils get a real fundamental grasp of mathematical principles. The distances we can go will depend to a large extent on how well our pupils are taught mathematics in your classes.

We hope you will not break faith with us by making your mathematics courses so easy they mean nothing. We also hope you will not be swayed by some educators who seem to believe that pupils will succeed better in this increasingly complex civilization of ours by knowing less mathematics. However, we do not believe you can justify the teaching of mathematics for the sake of mathematics alone. But we do feel there is a very necessary and fundamental place for mathematics in our schools if you can teach pupils how to use it as a tool and as a way of thinking in all fields of knowledge.

Mathematics Professional Barometer

The larger integer represents the number of primary and secondary teachers in both the private and public schools. The smaller integer represents the membership of the N.C.T.M. in the respective states. Increase the professional status of your state by getting at least one teacher to join the National Council!

"X on the Spot"

By Louise Meals
Evanston Township High School, Evanston, Illinois

Author's Note: The author realizes perfectly that the validity of some of the arguments used in the play could be questioned seriously. Readers are reminded, however, that all the statements used are actual quotations from persons not in the teaching profession and, as such, reflect the attitude of the layman toward mathematics.—L. M.

Characters

Dick Bradford, Jane Bradford, Bob Adams, Tim Brooks, Ann Montgomery — Pupils at the E.T.H.S.
Mrs. Bradford, mother of Dick and Jane

Time: *7 p.m., on a mid-September evening.*
Scene: *The Bradford living room. There is a davenport at the center back, with easy chairs at the left and right. A table stands at the left of the davenport, and there is a stool between the davenport and the chair at the right. A door at the rear left leads to the hall, and one at the rear right to the dining room.*

Scene I

Mrs. Bradford is on the davenport, knitting. Dick and Jane are sprawled in the easy chairs.

Mrs. B.: If you don't mind my curiosity, —why all this sudden interest in mathematics?

Dick: Well, y'see, Mom, we just *gotta* win the football championship this year!

Mrs. B.: (*Putting down her knitting and looking at him in a bewildered fashion*) Er—what?

Jane: What Dick means, Mother, is that if Bob Adams flunks math., he'll not be able to play football.

Dick: And if he's off the team—bingo! There'll go our chances for the championship!

Mrs. B.: But what has that to do with you two? Hasn't Bob enough backbone —he's certainly smart enough—to get down to business and pass the math.?

Dick: Sure he has, Mom! You don't understand! It's a matter of principle with him now, and even being kicked off the team won't budge him! He says he's not going to be an engineer, so why should he waste his time doing math.!

Mrs. B.: What *is* he going to do?

Dick: He doesn't know yet.

Mrs. B.: Then how does he know he won't need math.? Other people aside from engineers use it.

Jane: He knows that, Mother,—and he knows he has to have it to get into college, too, but he's just got stubborn over the whole thing. So we thought— Ann, and Dick, and I—

Mrs. B.: Where does Ann come in on this? She doesn't strike me as the type that would be especially interested in mathematics!

Dick: Heck, Mom! It's not math. any of us're interested in! It's getting Bob interested in it enough so he'll do the work and keep eligible!—But you're wrong about Ann. She gets nineties in math. all the time.

Mrs. B.: Then why doesn't *she* help Bob out?

Jane: Oh, Mother, that *would* be dumb! Why, she carries a red pencil around in her purse all the time, just so she can mark her papers down fifteen points before she shows 'em to Bob!

Dick: And, anyhow, that wouldn't help. We've gotta make him want to do it himself.

Mrs. B.: And just how are you planning to do that?

Jane: Well, we got Ann to organize a club to study careers—we knew Bob'd

join if Ann got the thing up—and for this first meeting we're going to study all the different careers where you use math.

DICK: And if it works—that is, if we can get Bob to do his math.—why, this will probably be the first, last, and only meeting of the famous Careers Club!

MRS. B.: What are all those papers you've got there?

JANE: Well—er—you see, we each interviewed some men and women around town and got them to tell us if they used math. in their work; and if they did, just how.

MRS. B.: (*Aghast*) Do you mean to tell me that you actually asked all those busy people to stop and do that for you?

DICK: Sure! Why not?

JANE: After all, Mother, it was in a good cause!

MRS. B.: How many of you are there in this club?

JANE: Oh, just us and Ann and Bob.

DICK: And Tim Brooks. Don't forget him! Bob made us ask him—they're always together.

JANE: And is he ever going to be a problem, tonight! He's awful smart in everything else—just a whiz in languages—but he's dumber'n dumb in math. He just can't see through it, that's all. (*Bell rings*) Oh, there they are, now!

Exit Jane—Left

MRS. B.: (*Folding up her knitting and rising*) Well, I'll leave you to your nefarious plans. There'll be something for you to eat in the dining room about half past nine, Dick. Remember, they can't stay late! Football championship or no football championship, this is a school night!

Curtain

SCENE II

Bob and Tim are on the davenport. Jane and Ann occupy the easy chairs, while Dick sits on the stool.

BOB: What I say is, why do we waste a whole evening talking about careers where you use *math.?* Why not English? There's some sense to that! You've just gotta know how to express yourself if you want to be successful!

ANN: (*Mildly*) But we all know that, Bob,—that's why we don't have to talk about it. What we do need, though, is to find out about the things that call for work in some particular subject.

TIM: But why start with math.? Why not languages?

JANE: Well, we took math. because so many things depend on it.

BOB: Oh, yeah?—What?

JANE: You remember that picture, don't you, that's in all the math. rooms? The one that was copied from the Science Building at the Chicago Fair?

DICK: You know the one she means, Bob. It shows a whole lot of different sciences as branches of the Tree of Knowledge, —and the root of the whole tree is mathematics.

BOB: Yeah, I know!—And I bet some math. teacher thought that up, too!

ANN: But, Bob, you *do* need math. for all those sciences!

JANE: Math. must be *awfully* important, Bob, for a lot of things, or the colleges wouldn't require so much of it for entrance.

BOB: Oh, wouldn't they, just! I betcha two bits they just require it because some guy a million years ago found out it was a swell way to keep a lot of people like me out of college!

ANN: Oh, well, let's stop arguing, and get down to business.—Did you all bring a list of the things you want to do? (*Tim gets out a list*) What're you going in for, Tim?

TIM: Well, I guess I'll be an architect. I could go to Paris and study,—'n' I like French 'n' ev'rything.

ANN: Got anything there about architects, Jane?

JANE: (*Running through the statements in her hand*) No, I'm afraid I haven't.

BOB: (*Triumphantly*) There—you see? You don't need math. for one of the very first things a fella'd be interested in!

DICK: Heck, Bob! We couldn't investigate *ev'rything* in two days! And, anyhow, it stands to reason an architect'd *have* to have some training in math.!

BOB: I don't see why. He could draw up the plans, and let some contractor worry about the actual construction.

ANN: But, Bob, how could he draw the plans in the first place?

JANE: And what if they weren't practical after that?

DICK: (*Sarcastically*) I suppose he could just keep on doing 'em over and over until he just *happened* to get 'em right! Seems like an awful waste of time, if that's what they actually do!

BOB: (*Thoughtfully*) Well, maybe you're right, at that.

ANN: You'd better cross that off, Tim, if you don't like math.!—What's next?

TIM: Well, if I can't be an architect, I guess I'll be an artist.—You don't need math. for that.

JANE: Well, I don't know about that, Tim. (*Takes up a statement and looks at it*) Miss Goffe says here that you have to use geometry all the time in design.

TIM: (*Ruefully*) Gosh, that's out, too! Well, I guess I'll have to be a diplomat. That's the only other thing I've thought about.

ANN: (*Enthusiastically*) Oh, that'll be fine, Timmy! I guess all diplomats have to do is to talk in seven different languages. I don't think (*judicially*) they use any math. At least, Dad says their reasoning isn't always very clear or logical.

JANE: Then maybe they'd do a better job if they did use some math.!

BOB: What beats me is why you girls have taken such a sudden interest in this darn stuff! You won't ever be using it!

JANE: That's what you think!—You know I'm going to be a librarian; and when we went in to the Public Library today to talk to Miss Wright, the very first thing she asked me was what courses I was taking that would test how accurate I was, and whether I could carry a thing through to completion. Of course, I said math., right off, and she seemed awfully pleased about that.

BOB: I don't see why.

JANE: Well, she said math. helped you to be accurate in keeping the records of the books that come in and go out,—and in all sorts of other things!

ANN: For instance, she says librarians have to do real detective work in tracing people and events from very slight clues; and that alertness and logical thinking are terribly necessary.

JANE: And she said she'd found that any student whose school records showed ability in math. could be counted on to do good work in a library. And—

BOB: (*Interrupting, hands up*) Hey, stop! I give in! *You need math. to work in a library!*

TIM: But what about you, Ann? You're not going to be a librarian, too, are you?

ANN: No. I'm going to have a tearoom, all my own. You know,—a very swanky place, all modernistic stuff, and ev'rything!

BOB: So I suppose you tackled Don Robertson, or someone like that!

ANN: (*Coolly*) Right, the first time! And he says they use math. in lots of places in the restaurant business.

BOB: Where?

ANN: Oh, in the buying end of it; and in figuring the income tax and the federal taxes of all sorts,—and the profit, if any!

BOB: But, gosh, Ann, that's just simple arithmetic! Ev'rybody's gotta be able to do that!

DICK: Oh, yeah?—Can you?

BOB: Well—no! But I can see some sense to learning *that!* It's algebra and geometry I'm agin!

ANN: But, Bob, you know we get all sorts of drill in arithmetic in all the math. classes, no matter what they are,

—and it's a lot more interesting to have it that way than it would be just to go over again the same stuff we had in the grades. You'll have to admit that!

TIM: Boy, she's got you there!

ANN: (*Hastily*) What about you, Dick? What are you going to do?

DICK: Well, I'd thought some of insurance.

BOB: There's no use arguing about that! You'll need all the math. you can get, if you're going anywhere in that!

JANE: Good for you, Robert, old scout! You'll really admit there's some good in studying math., after all!

BOB: (*Magnanimously*) Oh, sure, it's all right for anybody that really needs it!

DICK: I'm sorta interested in engineering, too. Only, I'm not sure yet just what kind, so I've been talking to some people around town, to try to get a line on things.

JANE: Didn't the engineers at the Public Service Company tell you they used formulas and graphs a lot?

DICK: Yeah,—all the time, in connection with the gas supply.

ANN: And Mr. Nethercott said he'd send you some material, didn't he, Dick,—about civil engineering?

DICK: He said he'd show me some of the technical formulas they use in working with the water and sewage systems.

TIM: Well, well, it looks as if those old formulas had a use, after all!

ANN: Honest, Tim, you'd never believe all the different places they use math.!

DICK: Optometry, for instance! Mr. Hattstrom told me you need two years of algebra, one of geometry, and one of trigonometry, to go in for that.

JANE: And don't forget what Mr. Dickinson says! (*Hands him a statement*)

BOB: What Mr. Dickinson?

DICK: Mr. Kenneth Dickinson, the assistant director of the Bureau of Accident Prevention.

BOB: Now, don't tell me you're going to ring in any math. there! My brother's on the Safety Council, you know!

DICK: Well, then, old kid, just take a look at this! (*Hands Bob the statement*) He says they use formulas to determine the speed of a car from the skid marks left at the scene of an accident; and also, to determine the speed of a car from its stopping distance.

BOB: (*Perusing the statement*) Gosh, that's interesting, isn't it? I heard Ted talking about that one day, but it went over my head, then.

ANN: It's your turn, now, Bob. What's on your list?

BOB: Well, my Dad wants me to be a doctor, but I'm not very keen about it. I wouldn't have to use math., though, and I'd like that.

DICK: Wait a minute young feller, me lad! Ev'ry time I go into Dr. Kappos's office for my hay fever shots, I see all sorts of graphs around. There's one that shows the pollen count from day to day. She puts one over the other, so you can compare the seasons from year to year.

ANN: And don't you need chemistry for medicine, Bob? And I know you need math. for chemistry!

JANE: (*Excitedly*) And, Dick, what was that Mother was reading to us the other night, from Alexis Carrell's *Man the Unknown?*

DICK: It was how some Frenchman had discovered a formula you could use to tell how long it would take a wound to heal, if you knew the surface of the wound, its age, and the age of the person.

TIM: Whew, man, they've sure got you!

BOB: Well, I don't want to be a doctor, anyway. What I really want to go into is banking. And I suppose you're all primed for me there!

DICK: Boy, are we! We've got a letter from Mr. Tomlinson at the State Bank and Trust Company. I won't read all of it, but just listen to this last paragraph: (*Reads*) "Such studies as solid geometry, higher algebra, and trigonometry, if really mastered, are of invaluable

assistance in developing the powers of reasoning, which must daily be applied to problems and policies in which tables or formulas can not be used.—All hail, then, to the good old mathematics!"

BOB: Gosh! And that fellow really knows what he's talking about, too. Well, it'll have to be done, I guess, much as I hate it!—Ann, how about helping me with my math., until I get going again?

ANN: (*Stammering*) Why—well—

BOB: Oh, I know you get swell marks in math. Your mother always shows my mother your report cards!

ANN: Well—

MRS. B.: (*From the door*) Come, children, before the cocoa gets cold! Bob—Tim—

Exit Bob and Tim.

Jane, Ann, and Dick silently shake hands and exit, singing "Down the Field the Blue and Orange."

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Does a Year's Exposure to Algebra Improve a Pupil's Ability in Arithmetic?

By Benjamin Braverman
High School of Commerce, New York, N. Y.

The National Committee on Mathematical Requirements in its epoch-making report issued in 1923 lists as one of the important practical aims of instruction in secondary mathematics, an improvement in a pupil's arithmetic ability. This was to be effectuated in at least three ways, only two of which need be mentioned here. The first was the attainment of accuracy and speed in numerical computation as a result of the pupil's experiences with the many numerical problems arising in the field of secondary mathematics. The second was a progressive increase in the pupil's understanding of the fundamental operations and the power to apply them in new situations because of his exposure to them in the field of algebra. No one, not even the general educator, will quarrel with the National Committee over the desirability of this aim. But are we achieving it? In particular, taking the most important branch of secondary mathematics, ninth year algebra, can we definitely point to an improvement in a pupil's arithmetic skill and knowledge as a result of his year's exposure to algebra? This is the question that this paper will attempt to answer in an objective way.

The study of a pupil's improvement in arithmetic as a result of his exposure to a year of algebra grew out of an arithmetic test which was administered to about 460 pupils in the second and third terms of the High School of Commerce in New York City for classificatory purposes in connection with a course in ninth year algebra they were about to take. Like all secondary schools of the country, the High School of Commerce is receiving a type of pupil whose previous preparation, whose ability for quantitative thought, and whose attitude toward learning in general makes it impossible to expose all pupils taking ninth year algebra to the traditional course of study. The school has, therefore, been confronted with two major problems; first, how to modify the traditional course to meet the needs and abilities of the new type of pupil; second, how to classify its pupils into those who were to take the traditional algebra course, called the *R* course in the High School of Commerce, and those who were to take the modified course, called the *B* course. It has tried to solve the first problem by preparing a course in algebra with emphasis upon algebraic symbolism, the formula, especially the formula in business relationships, functional relationships including graphs, the equation, the verbal problem, particularly those types frequently arising in business. It has tried to solve the second problem by experimenting with several classificatory devices. In September, 1935, it decided to experiment with an arithmetic test. The various standardized arithmetic tests on the market were not considered satisfactory for its purposes and so it prepared a test of its own.

This arithmetic classificatory test was in two parts. The first part contained 25 examples which tested such fundamental skills as the fundamental operations with integers and fractions including decimals and such fundamental concepts as per cent and square root. The second part contained 20 examples designed to test simple quantitative thinking. Both parts were in three forms. While no scientific test was made of the equivalence of the three forms, they were drawn up so carefully

* Report of a study made at the High School of Commerce, New York, N. Y., by Benjamin Braverman.

TABLE 1

RESULTS IN ARITHMETIC SKILLS
463 Pupils Examined

Example No.	Description of Example	No. Having Example Wrong	% of Those Taking Test	Rank of Example in Difficulty	Most Common Error in Solution of Example
1.	Add 127, 38, 579, 1591, 8	62	13	22	
2.	Subtract 798 − 346	4	1	25	
3.	Subtract 5372 − 986	35	8	23	
4.	Multiply 268 × 76	127	27	15	Adding incorrectly the partial products.
5.	Divide 49)1715	63	14	21	Subtracting incorrectly the partial products.
6.	Arrange in a column and then add. \$4.25; \$12.96; \$.38; \$.09.	67	14	20	Improper arrangement.
7.	Subtract 5.00 − 2.73	34	7	24	Omission of decimal point.
8.	Multiply 1250 × .045	113	24	17	Disregard of the decimal point in the product.
9.	Insert the decimal point in the proper place in the following division example. .035)52.5 (15; 35; 175; 175)	314	68	5	Disregard of the decimal point in divisor, dividend, or quotient.
10.	Divide .18)216	345	75	2	No attention to the decimal point.
11.	Add $\frac{1}{3}$, $\frac{1}{3}$	114	25	16	Adding the denominators.
12.	Subtract $\frac{3}{4}$ − $\frac{1}{2}$	159	34	9	Subtracting denominators as well as numerators.
13.	Find the value of $\frac{1}{2}+\frac{1}{4}$.	154	33	10	Adding denominators as well as numerators.
14.	Find the value of $\frac{3}{8}\div\frac{1}{2}$.	227	49	6	Inverting dividend instead of divisor.
15.	Find the value of $3\frac{1}{8}+2\frac{1}{4}$.	219	47	7	Multiplying instead of adding.
16.	Find the value of $1-\frac{1}{4}$.	146	32	11	Adding instead of subtracting.
17.	Multiply $\frac{3}{4}$ by 4.	138	30	14	Multiplying by the denominator.
18.	Multiply $2\frac{1}{2}$ × $2\frac{1}{2}$	346	75	1	Failure to multiply.

TABLE 1—*Continued*

Example No.	Description of Example	No. Having Example Wrong	% of Those Taking Test	Rank of Example in Difficulty	Most Common Error in Solution of Example
19.	Divide $\frac{3}{4}$ by 6.	322	70	4	Inversion of dividend instead of divisor.
20.	Write $\frac{1}{4}$ as a per cent.	107	23	18	Adding the per cent sign to $\frac{1}{4}$ or .25.
21.	Write .02 as a per cent.	187	40	8	Adding the per cent sign to .02 or 2/100, or 1/50.
22.	Write 47% as a decimal.	88	19	19	Confusing 47% with 4.7%.
23.	Write $33\frac{1}{3}$% as a common fraction.	144	31	12	Confusing $33\frac{1}{3}$ with $33\frac{1}{3}$%.
24.	Congress appropriated four billion, eight hundred million dollars for recovery. Write this amount in numbers.	139	30	13	Confusing million with billion.
25.	What is the square root of 64?	341	74	3	Ignorance of the concept of square root.

that there is no reason to believe that one form was in any way more difficult than another. A copy of Form *A* of the Arithmetic Skills test appears in Table 1 with some very interesting data showing the reaction of the pupils to each of the 25 examples in the test. It must be kept in mind that the figures given represent the results in the *three* forms of the test. A total of 463 papers were examined.

While the distribution of scores in this part of the test, given in Table 2, is not germane to our present study, the reader will find it interesting because of the illuminating light it casts upon the arithmetic preparation of the pupil whom we are now asked to instruct in secondary mathematics. The slight difference between the total number of cases in the two tables is due to discharges and transfers.

A copy of Form *A* of the second part of the classificatory test given September, 1935 appears in Table 3. The reader will remember that this part of the test emphasized the very simplest type of quantitative thinking. The reaction of the pupils tested to each of the 20 examples in this test also is given. A total of 466 papers were examined.

A distribution of the scores in this part of the test is given in Table 4 as further revealing the type of pupil found in our

TABLE 2

DISTRIBUTION OF SCORES
Arithmetic Skills

No. of Examples Solved Correctly	Frequency
0	
1	
2	
3	
4	
5	
6	3
7	4
8	6
9	15
10	14
11	23
12	22
13	33
14	30
15	35
16	53
17	32
18	39
19	40
20	36
21	18
22	19
23	7
24	6
25	1
Total No. of Pupils	436
Mean	16.0
Median	16.6

TABLE 3

RESULTS IN ARITHMETIC THOUGHT

466 Pupils Examined

Example No.	Description of Example	No. Having Example Wrong	% of Those Taking Test	Rank of Example in Difficulty	Most Common Error in Solution of Example
1.	What is the sum of 16 and 8?	16	3	18	Dividing or subtracting instead of adding.
2.	What is the difference between 10 and 7?	14	3	19	Dividing instead of subtracting.
3.	What is the product of 12 and 3?	152	33	6	Confusing product with sum or quotient.
4.	In a class of 35 pupils, 27 passed a certain test; how many failed?	12	2	20	Dividing instead of subtracting.
5.	If an aeroplane travels at the rate of 120 miles an hour, how far will it travel in 5 hours?	21	5	17	Dividing instead of multiplying.
6.	A man gives a cashier a $20.00 bill in payment of a purchase of $13.57. How much change should the man receive?	43	9	16	Omission of decimal point.
7.	An automobile covered the distance of 150 miles between New York and Albany in 6 hours. What was the average speed of the car?	46	10	15	Multiplying instead of dividing.
8.	A man's bank balance at the beginning of a day was $1420.75. During the day he issued the following checks; one for $25.00; one for $537.65; one for $8.75; and one for $69.81. There were no deposits made during the day. What was balance at the end of the day?	145	31	7	Some error in addition.
9.	A boy's marks in three history tests were 80, 85, and 78. What was his average mark?	60	13	12	Error in division.
10.	Find 6% of $750.	141	30	8	Finding $\frac{1}{8}$ of 750.
11.	If a dozen pencils cost 48 cents, how much will 30 pencils cost?	123	26	10	Changing 30 to an incorrect number of dozens.
12.	A man saves 20% of his income. If his income is $4500, how much does he save?	90	19	11	Some error in connection with decimal point.
13.	A dealer buys an article for $25.00. He sells it at a profit of 40% of the cost. What is the selling price of the article?	166	36	5	Finding the gain instead of the selling price.
14.	In a class of 36 pupils, 27 pupils passed a certain test, and 9 failed the test. What is the per cent of those who passed?	222	48	4	Using the wrong base.
15.	A man bought a typewriter for $90.00, and sold it at a loss of 10% of the cost. How much did he sell it for?	128	27	9	Confusing 10% with $10.00.
16.	A pound of butter costs 56 cents. Jane bought $\frac{3}{4}$ of a pound. How much should Jane pay the storekeeper?	52	11	13	Error in multiplication.
17.	A party of 15 boys contributed an equal amount to pay the cost of an outing, which amounted to $4.50. How much did each contribute?	51	11	14	Multiplying instead of dividing.

TABLE 3—*Continued*

Example No.	Description of Example	No. Having Example Wrong	% of Those Taking Test	Rank of Example in Difficulty	Most Common Error in Solution of Example
18.	A man paid \$1.50 for $\frac{5}{8}$ of a yard of cloth. At the same price, how much would he have had to pay for one whole yard of cloth?	321	69	1	Failure to get cost of $\frac{1}{8}$ of a yard.
19.	A man pays interest at the rate of 6% per annum on a loan of \$5000. How much interest should he pay Sept. 1, 1935 if the last interest payment was made March 1, 1935?	300	64	2	Incorrect time factor.
20.	A man has a $\frac{1}{4}$ interest in a business and gives $\frac{1}{3}$ of his share to his son. What is the son's interest in the business?	295	63	3	Failure to multiply the two fractions.

secondary schools today. Again the slight difference between the totals in Tables 3 and 4 is due to discharges and transfers.

TABLE 4

DISTRIBUTION OF SCORES

Arithmetic Thought

No. of Examples Solved Correctly	Frequency
0	
1	
2	
3	
4	1
5	3
6	1
7	5
8	11
9	9
10	27
11	20
12	27
13	44
14	45
15	58
16	52
17	58
18	50
19	31
20	9
No. of Pupils	451
Mean	14.6
Median	15.6

It was after the study of pupils' errors in the test administered in September, 1935 had been made that the idea suggested itself of determining whether the pupil's exposure to a year of algebra had had any effect upon his arithmetic knowledge and skill. Hence in June, 1936 the same arithmetic test in its two parts was administered to all pupils who had had a year's work in the traditional or modified algebra course. In every case, a pupil's scores in June, 1936 were compared with the same pupil's scores in September, 1935. In the tables which follow, therefore, the comparison is made between groups that are absolutely homogeneous. This is the reason that, although over 400 pupils took the test in September, 1935, the numbers in the distribution which follow are approximately 200. Many of the pupils who took the test in September, 1935 failed to take it in June, 1936, and vice versa, many of those who took it in June, 1936 had failed to take it in September, 1935. Also to insure a fair sampling, the test in June, 1936 was administered to those who had failed the first term of algebra and were repeating it, as well as to those who were completing the year's work in algebra. Of the approximately 200 in the distributions which follow, about 50 per cent represent pupils who completed the year's work in the modified course, about 30 per cent represent pupils who completed the traditional course, and about 20 per cent represent pupils who failed in the first term of the modified course and who were repeating the modified course.

Table 5 and Graph 1, which follow, give the distribution of scores in the Arithmetic

TABLE 5

ARITHMETIC SKILLS
Distribution of Scores Before and After Exposure to Algebra

Score	Frequency Before Exposure	Frequency After Exposure
1		
2		
3		
4		
5		
6		
7	2	
8	2	
9	8	
10	8	
11	11	
12	11	1
13	18	1
14	14	8
15	17	7
16	15	7
17	13	15
18	18	9
19	14	15
20	16	35
21	8	26
22	7	17
23	4	24
24	4	19
25		6
Total	190	190
Mean	15.8	20.2
Median	16.3	20.9
Standard Deviation	4.0	2.9

TABLE 6

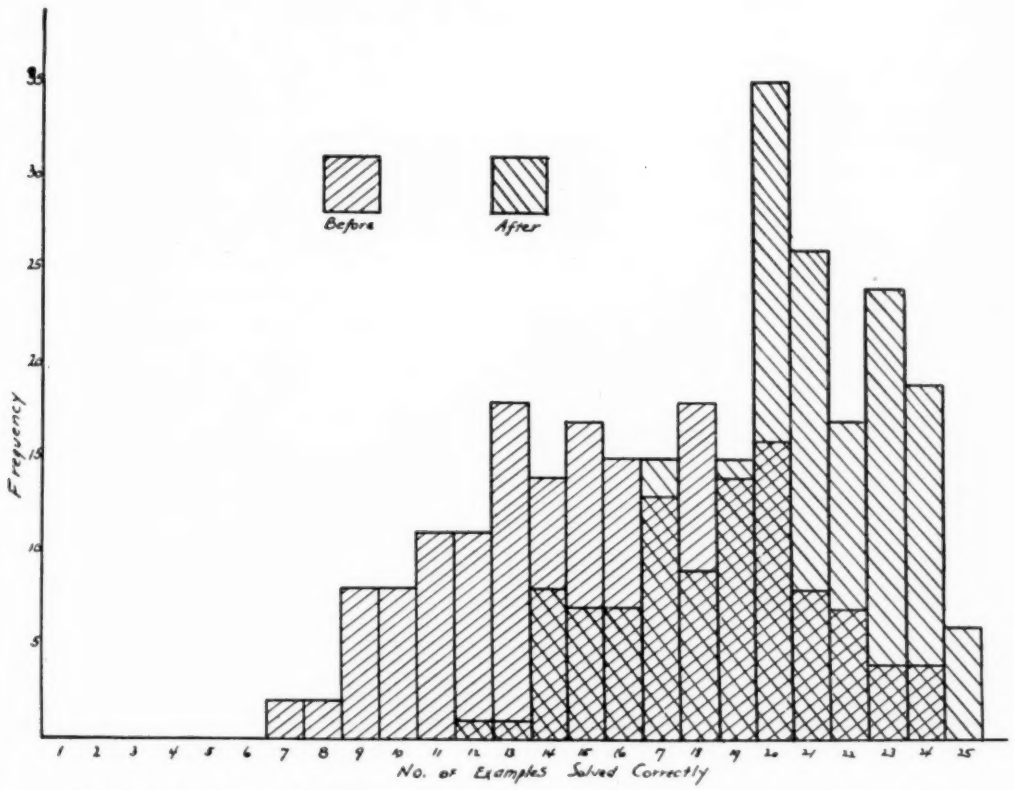
ARITHMETIC THOUGHT
Distribution of Scores Before and After Exposure to Algebra

Score	Frequency Before Exposure	Frequency After Exposure
1		
2		
3		
4		
5		
6	1	
7	4	1
8	4	1
9	4	
10	15	5
11	6	3
12	15	11
13	22	18
14	15	17
15	30	24
16	18	33
17	24	27
18	17	20
19	8	18
20	2	7
Total	185	185
Mean	14.3	15.6
Median	15.2	16.4
Standard Deviation	3.1	2.5

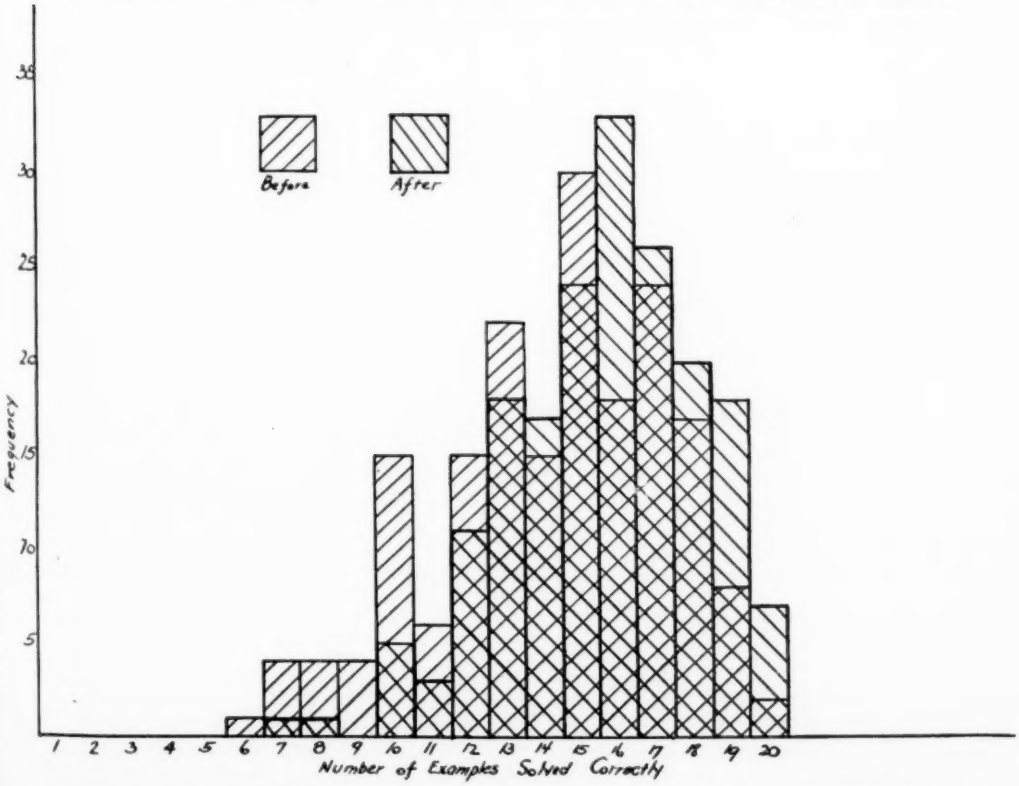
Skills part of the test of 190 pupils before exposure to algebra and after exposure to algebra. The reader will note that the mean and median of the distribution of these 190 cases before exposure to algebra is only slightly different from the mean and median of the 436 cases in the original distribution. The reader will also note a significant rise in the mean and median, the former showing an increase of 4.4 examples solved correctly, and the latter showing an increase of 4.6 correct solutions. Also, whereas the lowest score before exposure to algebra was 7, the lowest score after was 12. Finally, although none of the 190 pupils were able to score a perfect mark before exposure, 6 pupils succeeded in doing it after exposure.

Table 6 and Graph 2 give a similar comparison of the results in the Arithmetic Thought part of the test before and after exposure to algebra. It is interesting to note that while there is an increase in both the mean and median here as in Table 5, the increase in the former is only 1.3, and in the latter 1.2. However, while only 2 of the 185 pupils made a perfect score before exposure to algebra, there were 7 of the same pupils who made such a score after exposure.

It was deemed advisable to combine the scores in the two parts of the test both before exposure and after exposure and to compare them. Table 7 and Graph 3 give such comparison. Only 170 cases are considered in this table because all pupils who did not take both tests in September, 1935 and June, 1936 had to be eliminated. The reader should note again the increase in the mean amounting to 5.9 and in the median amounting to 6.1; also that whereas the lowest score before exposure was 13, the lowest score after exposure



GRAPH 1. Arithmetic Skills. Distribution of scores before and after exposure to algebra.



GRAPH 2. Arithmetic Thought. Distribution of scores before and after exposure to algebra.

was 25; finally that the highest score before exposure was 42, whereas after exposure there were 8 pupils who scored a higher mark than 42.

TABLE 7

ARITHMETIC SKILLS AND THOUGHT COMBINED
Distribution of Scores Before and After Exposure to Algebra

Score	Frequency Before Exposure	Frequency After Exposure
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13	1	
14		
15		
16	2	
17	2	
18	2	
19	1	
20	6	
21	4	
22	4	
23	5	
24	10	
25	5	2
26	8	4
27	8	3
28	14	8
29	5	4
30	9	8
31	6	5
32	8	4
33	12	9
34	12	6
35	7	12
36	10	20
37	8	7
38	3	22
39	5	12
40	6	13
41	4	17
42	3	6
43		6
44		1
45		1
Total	170	170
Mean	30.1	36.0
Median	30.9	37.0
Standard Deviation	6.4	4.7

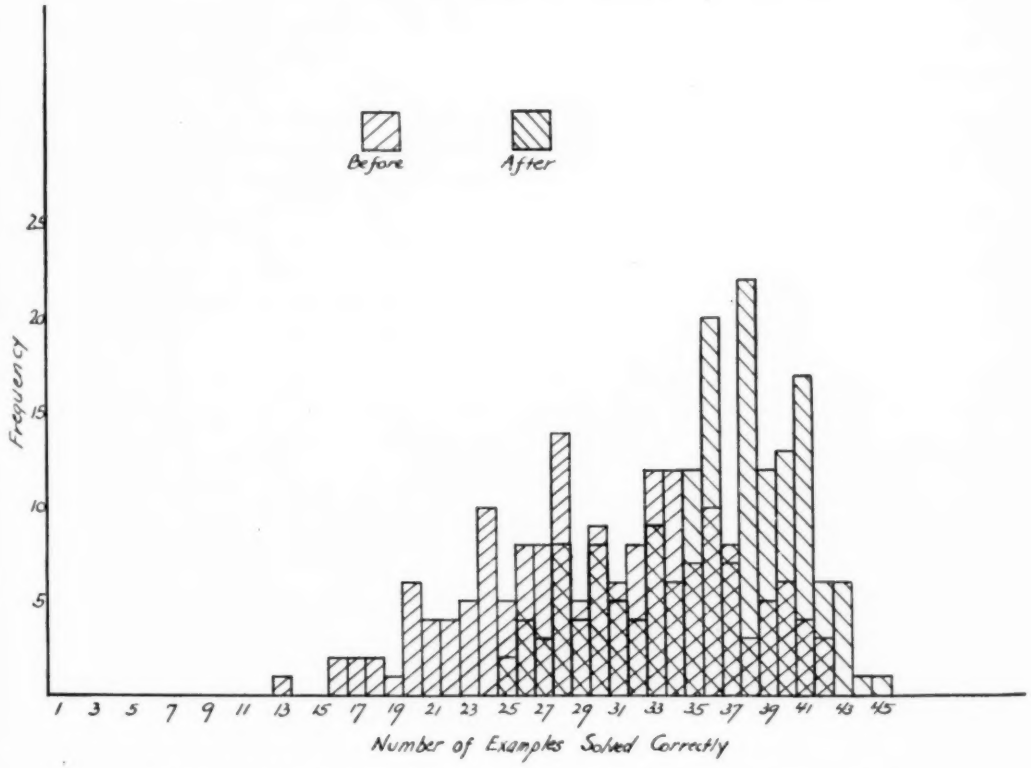
The three preceding tables while revealing a more or less satisfactory improvement in the arithmetic skill and knowledge of the entire group do not appear to cast very much light upon individual improvement. For that reason a study was made of the number of additional examples each pupil was able to solve correctly after exposure to the one year of algebra, with some very interesting results. Table 8 gives the results of such study for the Arithmetic Skills test. It will be noticed first that not all showed improvement; 12 pupils or 6 per cent of the total showed no improvement at all; and another 12 pupils even showed retrogression. However 166 pupils or 88 per cent of the total did show improvement, 4 pupils showing an improvement of 13 additional examples solved correctly. The reader

TABLE 8

ARITHMETIC SKILLS
Absolute Improvement After Exposure to Algebra

Number of Additional Examples Solved Correctly	Frequency
13	4
12	2
11	3
10	5
9	9
8	12
7	9
6	24
5	25
4	19
3	20
2	19
1	15
0	12
−1	4
−2	4
−3	4
Total Number Showing Improvement	166
Total Number Showing No Improvement	12
Total Number Showing Retrogression	12
Total Number Investigated	190
Mean	4.8
Median	5.2

Note: Those showing retrogression were not considered in computing the mean and median.



GRAPH 3. Arithmetic Skills and Thought Combined. Distribution of scores before and after exposure to algebra.

should note that in the computation of the mean and median of the distributions in Tables 8, 9, and 10, those showing retrogression were not included. It is for this reason that the mean and median of these distributions are slightly higher than the mean and median differences of the distributions in Tables 5, 6, and 7.

Table 9 gives the results of the same study for the Arithmetic Thought part of the test. It will be noticed here that the number showing no improvement at all or retrogression is much higher than in the Arithmetic Skills part of the test. In the former case, there are 35 pupils or 19 per cent of the total. In the latter, there are 39 pupils or 21 per cent of the total. However, 111 pupils or 60 per cent of the total do show improvement, 3 pupils showing an improvement of at least 10 additional examples solved correctly.

Table 10 gives the results of the same study for the entire test as a whole. The number showing no improvement at all is 10, or 6 per cent of the total. The number showing retrogression is 13, or 8 per cent of the total. The number showing improvement is 147, or 86 per cent of the total. Again the reader should note the surprising improvement shown by some of the pupils.

While the three preceding tables cast considerable light upon the improvement or lack of improvement shown by the individual members of the group, they fail to show the degree of improvement achieved by each individual. Thus a pupil who solved 10 examples out of 25 before exposure to algebra and 15 examples after exposure was placed in the same class in those tables with the pupil who solved 20 examples out of 25 before and 25 examples out of the 25 examples after. But obviously there is a radical difference between

TABLE 9

ARITHMETIC THOUGHT
Absolute Improvement After Exposure to Algebra

Number of Additional Examples Solved Correctly	Frequency
11	1
10	2
9	
8	
7	2
6	4
5	9
4	14
3	26
2	27
1	26
0	35
−1	16
−2	14
−3	7
−4	1
−5	
−6	1
Total Number Showing Improvement	111
Total Number Showing No Improvement	35
Total Number Showing Retrogression	39
Total Number Investigated	185
Mean	2.2
Median	2.4

Note: Those showing retrogression were not considered in computing the mean and median.

the two pupils. For the first has made only a $33\frac{1}{3}$ per cent improvement, while the second has made a 100 per cent improvement. A study was therefore made of the relative improvement shown by each pupil in each part of the test, with some rather interesting results. Table 11 gives these results in the case of Arithmetic Skills. In each case, the relative improvement was reduced to a per cent by dividing the number of additional examples solved correctly by a pupil, by the maximum number of examples that such pupil could have solved correctly. It will be noticed that 6 pupils showed 100 per cent improvement, that the mean of the distribution is 51.2 per cent improvement, with the median slightly higher or 54.1 per cent improvement. Again it should be borne in mind that the pupils showing retrogression were not considered in calculating the mean and median of the distributions in Tables 11, 12 and 13.

Table 12 gives the results of the same study in the Arithmetic Thought part of the test. While there are again 6 pupils who showed a 100 per cent improvement, the reader will not be surprised to learn

TABLE 10

ARITHMETIC SKILLS AND THOUGHT COMBINED
Absolute Improvement After Exposure to Algebra

Number of Additional Examples Solved Correctly	Frequency
23	1
22	
21	
20	
19	1
18	
17	3
16	1
15	4
14	6
13	3
12	5
11	5
10	7
9	10
8	9
7	13
6	20
5	9
4	11
3	13
2	15
1	11
0	10
−1	6
−2	4
−3	1
−4	2
Total Number Showing Improvement	147
Total Number Showing No Improvement	10
Total Number Showing Retrogression	13
Total Number Investigated	170
Mean	6.5
Median	6.5

Note: Those showing retrogression were not considered in computing the mean and median.

TABLE 11

ARITHMETIC SKILLS
Relative Improvement After Exposure to Algebra

Per cent of Improvement Possible	Frequency
100	6
90–99	6
80–89	12
70–79	21
60–69	22
50–59	37
40–49	18
30–39	9
20–29	24
10–19	8
0– 9	3
0	12
Retrogression	12
Total	190
Mean	51.2
Median	54.1

Note: Those showing retrogression were not considered in computing the mean and median.

that the mean of this distribution is only 36.6 per cent and the median 35.2 per cent.

Table 13 gives the results of the same study for the entire test. The reader will note there is only 1 pupil now showing 100 per cent improvement. The mean of this distribution is 42.2 per cent and the median is 43.2 per cent.

TABLE 12

ARITHMETIC THOUGHT
Relative Improvement After Exposure to Algebra

Per cent of Improvement Possible	Frequency
100	6
90–99	1
80–89	2
70–79	6
60–69	11
50–59	16
40–49	21
30–39	21
20–29	21
10–19	6
0– 9	
0	35
Retrogression	39
Total	185
Mean	36.6
Median	35.2

Note: Those showing retrogression were not considered in computing the mean and median.

TABLE 13

ARITHMETIC SKILLS AND THOUGHT COMBINED
Relative Improvement After Exposure to Algebra

Per cent of Improvement Possible	Frequency
100	1
90–99	
80–89	6
70–79	9
60–69	20
50–59	24
40–49	27
30–39	20
20–29	19
10–19	18
0– 9	3
0	10
Retrogression	13
Total	170
Mean	42.2
Median	43.2

Note: Those showing retrogression were not considered in computing the mean and median.

SUMMARY OF STUDY

The writer is aware that several serious objections can be raised from the scientific point of view to the study just made. No control group was used. Hence, the improvement shown might be attributed to the greater maturity of the pupils tested. Also the results in the second administration of the test were obtained after immediate exposure to some kind of quantitative course of study, whereas the first administration of the test occurred when some of the pupils had been away from arithmetic one year and most of them a half year. These objections are valid and are unanswerable. The reader should bear in mind, however, the great difficulty, one might even say, the impossibility of conducting truly scientific studies in the field of education.

However lacking, therefore, the study may be in scientific character, it does definitely point to one conclusion, and that is that after a year's exposure to algebra,

there is an improvement in the pupil's ability in arithmetic. Perhaps, the improvement shown by this study is not as great as we should like to have it. Other studies along the same line may reveal greater improvement or perhaps less, or even none at all. But until such studies are made leading to different conclusions, the teacher of algebra may well be cheered by the fact that his work appears to bear fruit in at least one respect, and that is, that an important objective in the teaching of algebra is being achieved. Of course, the cynic will say, why algebra to improve a pupil's ability in arithmetic? If our secondary pupils are deficient in arithmetic, would it not be better to give them that in which they are deficient? But the superior teacher knows that after eight years' exposure to arithmetic, it would be folly to give pupils more formal arithmetic. The pupil's deficiencies in arithmetic can be made up much better informally by emphasizing those types of examples which require the arithmetic skills in which pupils coming to our secondary schools, as this study revealed, are so woefully weak. Thus the skill for multiplying mixed numbers could be developed by an example like the following: If $A = s^2$, find A when $s = 2\frac{1}{2}$. Also the skill for dividing a fraction by a whole number could be developed by examples like the one: Solve the equation: $6x = \frac{3}{4}$. But is there need for further illustration? After the fundamental skills of the algebra have been mastered, the superior teacher realizing his pupils' serious arithmetic deficiencies, will use for further drill examples requiring those arithmetic skills in which pupils are deficient, rather than examples requiring more difficult algebraic manipulations.

Mathematics and Dictatorships

An interesting sidelight on the immunity of mathematics from the suppression of individual political freedom under authoritarian domination is given by E. E. Kellett in his "The Story of Dictatorship" (E. P. Dutton, 1937). This British writer refers, for example, on pages 186–7, to a characterization of the despotism of Napoleon by the German historian Heinrich von Sybel, as follows:

> External trade was of course extinct: in its place industry was devoted to such luxuries as silk, gold and glass. Where it was still possible to employ the labouring classes, their energies were set to the making of roads, houses or imposing buildings, remarkable rather for their colossal size than their beauty. *As for the sciences, the mathematical flourished, but history and philosophy were dead: and the reason is plain: they were the victims of the suppression of individual independence and personal liberty.* (Italics ours.)

In the concluding chapter (p. 208) Kellett comments further on the nature of life and thought under a dictatorship. Assuming, he suggests, an omniscient and morally perfect despot, managing a passive people, the question may well be asked: what sort of people will develop under such submissive regimentation? Their intellectual activities are likely to be limited to a few sciences or arts, such as are least dangerous to the ruler.

> "I well remember," he says, "observing in Russia, after the suppression of the revolt of the Intelligentsia in 1905, how these most highly cultured people, denied the chance of influencing politics, devoted themselves to *music, painting, or mathematics.*"

Events have moved rapidly in the last few years, and one wonders whether even science and mathematics under the third Reich will be so fortunate as to flourish unhampered in the limpid atmosphere of liberal scholarship.

—William L. Schaaf

The Relation of Mathematics to the Learning Process

By J. A. LYNCH
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THE purpose of this short paper is to suggest to the intuitions of the reader a concept of the relation between mathematics and the learning process which is based upon a hypothesis, worked out by the writer over a period of several years. As a hypothesis, it is of course open to criticism and possible revision.

This hypothesis represents the relationship between the *learning product* and its objective conditions (the learning situation or *stimulus*) by the following equation:

$$\sum_{i=1}^{i=n} a_i = A_n,$$

in which the left-hand member represents quantitatively the array of learning situations leading to A, and A_n stands for the learning product corresponding to the particular n terms or practices in the array.[1]

The term, a_i, is conceived as the objective or environmental counterpart of an insight *multiplied by its complexity*. There are two factors involved in a_i:

where, a_2 = "Complexity" or distinguishable "Parts,"
$(a_1 - a_2)$ = The "whole," minus its parts,
$a_i = [a_2(a_1 - a_2)]_i$.

The $(a_1 - a_2)$ factor of the right-hand member reflects the "Whole-minus-its-parts" concept, taken from the idealistic tradition by the so-called *Gestalt* psychologists; and the product, $a_2(a_1 - a_2)$, is essentially the "unity-times-complexity" concept taken from the same tradition by the writer.

[1] Lynch, J. A., "A Quantitative Description of the Stimulus," *Psychometrika*, Vol. 3, No. 2, June, 1938.

Now,

$$\sum_{i=1}^{i=n} a_i = \sum_{i=1}^{i=n} [a_2(a_1 - a_2)]_i.$$

As the learning-product, A, generalizes, the factor, a_i, expands in two directions:

1. Toward greater *inclusiveness*,
2. Toward greater *clarity*.

This implies that both factors, a_2 and $a_2(a_1 - a_2)$, should be increased simultaneously and without limits if such a thing were possible without confusing the learner.

Since an *insight* matures gradually, in relation to many practice situations, the amount of energy which goes into a_i should be, on the average, approximately equal to that which goes into the next practice, $a_{(i+1)}$. Therefore, the amount of energy which the *insight* commands at the a_i stage of its development necessarily sets a limitation on the product, $[a_2(a_1 - a_2)]_i$.

That is, a_2 can be large and $(a_1 - a_2)$ small, or *vice versa;* but they cannot both be large simultaneously *beyond the limits set by the psychic energy available at that stage.*

This difficulty can be counteracted and in some degree overcome by alternating, or perhaps intermingling, a_i's bearing the one characteristic with those bearing the opposite characteristic. This means that a_i's possessing a high degree of relatedness with respect to the number of *elements* contained in them should appear at intervals in the array, approximately alternating with those bearing the opposite characteristic.[2]

The repetition represented by the array, $\sum_{i=1}^{i=n} a_i$, is of *form* or *meaning* rather than

[2] *Memory* bridges the gap between the *schematic* and the *unschemed* practice situations until the learning product reaches maturity. This defines the role of memory in relation to the learning process.

of elements.[3] This being the case, it is apparent that the *forms* reflected by both types of a_i's must be either identical or analogous except in degree of clarity. The energy in one instance is expended in *clarification* at the expense of content, at the other in *spread* or in *inclusive grasp* at the expense of clarity.

Now, in certain kinds of learnings, notably those in which rationality is the form which psychic energy assumes, mathematics defines the *clarity* pole of this essentially bi-polar process of learning.[4]

The necessary alternation of opposite types of a_i's can be described as the *oscillatory* aspect of learning. Such *oscillations* are, of course, possible over either large or small cycles. It is possible to *learn*, then, without swinging as far in the direction of clarity as mathematics implies; but the maximum degree of *spread* is not provided for in such cases.

This reflects, to a certain extent, the argument attributed to J. F. Herbart[5] that mathematics is the tool of apperception.[6] It affirms Herbart's point that mathematics accelerates the process of learning from *nature*, and that it should be closely related to that process. The only limitation set upon mathematics by the present discussion is that it should be used to reflect clearly the patterns of meaning which are in the process of being projected into supplementary, *complex* situations. Also, the converse requirement holds for these complex situations in relation to the process of learning from *mathematics;* that is, the former should magnify and expand toward *inclusiveness* the meanings which are represented *with clarity* by the accompanying mathematical formulae.

This is also one sense in which mathematical training *transfers*[7] to other disciplines, notably to physics and chemistry, but to all other investigations of *nature* as well. To a certain extent, in the biological sciences and in geography *schematic drawing* and *maps* supplement and in some cases replace mathematical formulae and geometrical figures. Those who have the ability to schematize, which is cultivated by the study of mathematics, can obviously think more clearly about natural phenomena than those who lack it. If one is skeptical about the alleged connection between mathematics and *the process of learning from nature*, let him try to read "The Universe Around Us," by Sir James Jeans.[8] With a fair, general knowledge of mathematical methods, he can follow the writer of this book through a series of rapid intuitions into nature, as he applies the ever-lengthening measuring rods to the objects of outer space; without this knowledge, he is as a blind man trying to interpret the tales of those who see.

While the above argument does not rest upon the *practical* or functional value of mathematics, it is worthwhile to refer to the other side of Herbart's argument to the effect that *application* or control over nature branches off from the mathematical formulation of the concept being ap-

[3] If the learning product aimed at is a perceptual image, the factor repeated (the axis of the array) may be a sensory element or pattern; if a "value attitude" is aimed at, the *axis* of the stimulus is then defined by a series of similar or mutually-re-enforcing *goal situations*.

[4] As a matter of fact, this covers all sorts of schematizations for purposes of clarity of which mathematical *schema* are the most typical.

[5] Lynch, J. A., "Mathematics in Herbart's Philosophy of Education," THE MATHEMATICS TEACHER, Vol. XXXII, No. 2, Feb., 1939.

[6] An interesting contrast comes out in the comparison of J. F. Herbart and Immanuel Kant whose philosophy of spirit Herbart criticized. Kant, taking the *clarification from within approach*, linked geometry and arithmetic with *perception*, as what he called, "the pure forms of perception," or "perceptions *a priori*"; whereas, Herbart, taking the position that both the *content* and the *forms* of experience are determined from without, associated mathematics with *apperception*, since with him the latter takes the place of *perception* in the Kantian philosophy. *Space* and *time* also bear the same relation to apperception in Herbart's philosophy that they bear to perception in both the Leibnizian and the Kantian systems. The association psychologists arrived at a position similar to Herbart's and for similar reasons.

[7] Lynch, J. A., "The Problem of Transfer." *The Journal of General Psychology*, Jan., 1939.

[8] Sir James Jeans, *The Universe Around Us*, Macmillan Co., 1929.

plied. Any idea or principle not exactly formulated is difficult to *apply* effectively. The best answer to the query, "Will I ever have any use for mathematics?" is that "We use what we have mastered." Those numerous martyrs, who "studied mathematics" to no end, never learned any of it, to the point of mastery, or else it would have become a part of their thought mechanism. The mathematics advocate who makes only the *practical* appeal does not do his client full justice. The principal case for mathematics rests upon the twofold argument: first, as the science of form, mathematics makes a direct, aesthetic appeal to the human mind as its *object;* and, second, as one "pole" of the stimulus to learning, it bears a very direct relationship to what is thought by some philosophers to be life's most fundamental process. In fact, a large part of the idealistic tradition in philosophy can be accounted for as emphasis either upon the aesthetic appeal of pure *form*, making *mind* substance; or, upon the problem-solving and creative kind of activity, with the *learning* process conceived as substantial. Plato in ancient times is an example of the former, and John Dewey in the modern world represents the latter tendency.

No SCHOOL subject, except perhaps Classics, has suffered more than Mathematics from the tendency to stress secondary rather than primary aims, and to emphasize extraneous rather than intrinsic values. As taught in the past, it has been informed too little by general ideas, and instead of giving broad views has concentrated too much upon the kind of methods and problems that have been sometimes stigmatized as "low cunning." It is sometimes utilitarian, even crudely so, but it ignores considerable truths in which actual mathematics subserves important activities and adventures of civilized man. It is sometimes logical, but the type and "rigour" of the logic have not been properly adjusted to the natural growth of young minds. These defects are largely due to an imperfect synthesis between the idea that some parts of mathematics are useful to the ordinary citizen or to certain widely followed vocations, and should therefore be taught to everybody, and the old idea that, when mathematics is not directly useful, it has indirect utility in strengthening the powers of reasoning or in inducing a general accuracy of mind. *We believe that school mathematics will be put on a sound footing only when teachers agree that it should be taught as Art and Music and Physical Science should be taught, because it is one of the main lines which the creative spirit of man has followed in its development. If it is taught in this way we believe that it will no longer be true to say that "the study of mathematics is apt to commence in disappointment," and that it will no longer be necessary to give the number of hours to the subject that are now generally assumed to be necessary.*

Report of the Consultative Committee on Secondary Education with Special Reference to Grammar Schools and Technical High Schools. London, His Majesty's Stationery Office.

Popularizing the Subject of Astronomy*

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As an introduction to this brief discussion of the topic, "Popularizing the Subject of Astronomy," I should like to quote a statement of the Premier of all American mathematical thinkers and writers, Dr. David Eugene Smith. He said:

> Instead of the old educational attitude, which demanded that a student should accept the offering or leave it, modern education like modern hygiene seeks to make its waters both potable and health-giving.[1]

In other words, both palatable and digestible, interesting and comprehensible.

For several years and on numerous occasions, I have tried to emphasize my deepening conviction that the mathematicians, or probably I should say the mathematics teachers, have done more to discredit the subject in the minds of students and the general public than all the unseasoned criticism of the modern curriculum reformers.

Some have suggested with air of superhuman widsom that there is no such thing as eternal verity,—absolute fact, truth, or principle;—that facts, truths, and principles have only a relative existence; that there exists no such thing as static fact, truth, or principle; that such concepts are necessarily progressive; that what was considered fact, truth, or principle yesterday, last year, a decade, or a century ago, may appear in an entirely different light when considered in relation to the changed conditions of today; and likewise, what we consider facts, truths, or principles today, may undergo a complete change in order to fit the new conditions and needs of tomorrow, next year, a decade, or a century hence. To my way of thinking, such a line of sophisticated babbling is just as silly and ridiculous, and far more dangerous, than to contend that "*twedledee*" has had a more far-reaching influence on the amelioration of mankind and the progress of civilization than "*twedledum.*" Such an attitude prompts one to substitute opinion for fact, or appearance for established evidence. It was just as true that the earth and other planets revolved around the sun in the time of Ptolemy as in the time of Copernicus; however, Ptolemy based his System of the Universe upon appearances, or opinions, while Copernicus drew his conclusions from proven facts.

Strange and savage as it may seem, it is nevertheless true that men, from the early dawn of human history even down to the present time, have estimated their superiority over their fellows upon the basis of their ability to convince others that they have discovered and mastered hidden knowledge which no one else ever conceived.

The unpopularity of mathematics as a school subject in our course of study today has been brought about more as a result of the deliberate efforts of the so-called mathematicians to veil its facts and principles behind a dark curtain of mystery than all other causes combined. As proof of this statement, let me quote from Dr. Hogben's recent book, *Mathematics for the Million:*

> The pupils of Pythagoras formed themselves into secret societies and invested mathematics with the august and awful mystery *which it has had ever since.* Mathematical discoveries were communicated under oath. In the fourth century B.C., Hippasus, a member of one of these societies, is said to have been drowned in his bath for giving away mathematical truths for nothing.[2]

* A paper read before the Texas Academy of Science at Waco, Tex., on Friday, November 11, 1938.

[1] Sanford, Vera: *A Short History of Mathematics*, Houghton Mifflin Company, Boston, 1930; pp. iii–iv.

[2] Hogben, Lancelot: *Mathematics for the Million*, W. W. Norton and Company, Inc., New York, N. Y., 1937; p. 190.

Instead of simplifying, rationalizing, vitalizing, and practicalizing mathematical facts, principles and methods, the so-called mathematicians have succeeded in driving the great mass of students from it by the impression gained by them that the subject is too dark, mysterious, dry, uninteresting, useless, and of no practical value for the average mind to waste its time in the attempt to comprehend and to unravel its mysterious riddles.

To my way of thinking, for one to assert that one particular subject is interesting and another uninteresting is simply another way of saying that one has become familiar with the facts and their applications of the one subject while ignorant of the other. Enjoyment is found in the things which we have mastered by long continued effort and practice. This is especially true of the worthwhile things of life. Interest of the student does not depend necessarily on the type or sphere of any particular subject, *per se*. The subject of astronomy may become just as interesting and as full of beauties and values as any other subject, provided the necessary conditions are met. Students, most of them at least, learn to like the subject if they like the teacher. With the possible exception of a few of the most brilliant, students don't give a "continental" about what the teacher thinks he knows about the subject, if that teacher makes himself disagreeable and repulsive by his "hardboiled" arrogance and self-conceited egotism.

In the light of these observations, let us see just how the subject of astronomy may be popularized.

It should be remembered that astronomy is the oldest of all the sciences. It may be called the mother of science. The beauties and grandeurs of the heavens have inspired the wonder, the admiration, and the most anxious and serious thought of the historian, the philosopher, bard, and artizan since the early dawn of human history. The simple shepherds who guarded their flocks on cloudless nights upon the Chaldean hills and along the fertile plains and valleys of the Euphrates and the Nile were enraptured with joy as they gazed into the limitless vault of the skies, and catalogued the number and gave names to the stars and constellations as they were known to the ancient world.

Gradually emerging from the long night of superstitition and mysticism of the primitive tribes, through the twilight dawn of the wakening astrologers, the subject of astronomy has advanced to the position of exactitude alongside the subjects of physics, chemistry, geology, and biology. The hypotheses which constitute the foundation upon which the science of modern astronomy rests are just as secure and dependable as those of any of the other sciences, with the possible exception of pure mathematics.

The modern astronomer groups his conclusions under the following three classes: 1. those supported by such an abundance of observational and deductive evidence as may be accepted as true, such as the shape, size, and movements of the earth; causes and frequencies of eclipses, etc.; 2. tentative hypotheses, such as the existence of other undiscovered planets, causes determining the changes and nature of climatic conditions, etc.; and 3. hopeful speculations,—those on the border of the unknown, such as the origin and evolution of the planets, the existence of life on other heavenly bodies, etc. All great astronomers classify their conclusions under these three groups so that we may not be left in the dark as to what may be considered facts, theories, or speculations. This fact really distinguishes the difference between astrology and astronomy; the former based upon superstitious and imaginative conclusions formulated to suit the selfish whims and desires of individuals; the latter based upon observation, experimentation, and careful checking of facts and testing of clearly obvious assumptions.

If the student is made familiar with the facts and principles underlying the simple

mechanics of the heavens, the subject of astronomy may become one of the most fascinating studies that engage the attention of the human mind. This may be accomplished by means of a list of choice books on the subject, original drawings and charts for illustrations, astronomical slide pictures, frequent observations of the heavens with a telescope, and by regularly keeping on the outlook for interesting news items on astronomical facts and discoveries in the leading newspapers and magazines.

The more important of these facts and principles pertaining to the simple mechanics of the heavens may be summarized briefly as follows:

1. The sun is the center of the Solar System, around which revolve the nine planets and the eight hundred to one thousand planetoids in counter-clockwise direction, and in almost the same plane.

2. Using the distance from the earth to the sun as the *astronomical unit* of measurement, the numbers representing the approximate distances of these planets from the sun may be remembered easily by means of a simple device, known as Bode's Law, as 0.4, 0.7, 1.0, 1.6, 2.8, 5.2, 10.0, 19.6, 38.8, 77.2; approximately correct, except for the last two. (Nearly right: *30.0, 39.6*)

3. That the orbits of the planets are elliptical in shape, with the sun as one of the foci.

4. That the radius vector of any planet sweeps over equal areas in equal intervals of time.

5. That the squares of the periods of any two planets are to each other as the cubes of their mean distances.

6. That none of the planets are self-luminous, but shine only as reflected light from the sun.

7. That an eclipse of the sun, moon, or of any of the planets, is caused by the passage of some other heavenly body so as to shut off the light of the sun.

8. That the rotation of the earth on its axis with reference to the sun produces the day, and its revolution around the sun together with the inclination of its axis with the perpendicular to the plane of its orbit, produces the seasons and the year.

9. That the rotation of the earth on its axis causes the sun, moon, planets, and stars to appear to rise in the east and set in the west.

10. That the limitless space of the universe in every conceivable direction from the Solar System contains innumerable other stars, or suns, which may be considered individually or in groups, so far away that their movements produce no appreciable change in their relative positions; hence, are called *Fixed Stars.*

11. And then a clear understanding and familiarity with a list of the more common astronomical terms, or what might be called an astronomical vocabulary, some of which may be grouped as follows:

Those used to designate positions and to locate heavenly bodies: Altitude and azimuth; right ascension and declination; zenith and nadir; aphelion and perihelion; apogee and perigee; horizon, orbit, ecliptic, milky way, celestial sphere, celestial equator, celestial meridian, celestial poles, parallels, equinoxes, solstices, zodiac.

Those used as units of distances: Astronomical unit, light year, parsec.

Those used to distinguish types of heavenly bodies: Asteroid, planetoid, satellite, moon, planet, star, sun, meteor, comet, nebula, meteorite, system, constellation, galaxy, supergalaxy, etc.

Those used to indicate divisions of time: Natural units of time, as day, season, year; artificial units of time, as second, minute, hour, week, month; kinds of days, as astronomical, civil, solar, lunar, sidereal; kinds of years, as exact or true, tropical, legal, fiscal, Gregorian, Julian, common, leap.

Again, it should be remembered that astronomy is a live, growing subject. The widespread interest in almost every state and nation in the construction of huge telescopes and the erection of expensive astronomical observatories is evidence of the serious consideration that is being

given this field of knowledge and discovery.

A list of the more important astronomical observatories compiled in the year 1930 showed twenty-three in Europe, twenty-two in North America, five in South America, five in Asia, and four in Africa and Australia. Since 1930, there have been four large telescopes constructed in North America: one at Harvard University, Massachusetts; one at Perkins University, Ohio; one at the Naval Observatory, Washington, D. C.; and one at the University of Toronto, Canada. Two other great observatories are being completed in the United States: the W. J. McDonald Observatory of the University of Texas on Mt. Locke near Fort Davis, to contain an 82-inch reflector; and the other on Mt. Palomar, south of Pasadena, California, which will house the giant 200-inch reflector, the largest in the world.

As another evidence of the widespread interest in the subject of astronomy, the leading newspapers and magazines frequently contain interesting news items concerning astronomical facts and discoveries. A single instance may be cited. On October 18, 1938, there appeared in the *Dallas Morning News* an article from the United Press relative to the McDonald Observatory of the University of Texas, stating that "the 82-inch, three-ton mirror was ready Monday night after four years of polishing for a coating of aluminum so that it can show man what lies more than 900,000,000 light years from the earth." On the same date, the *Dallas Times Herald* carried a lengthy article relative to the recent discovery of the two new moons of Jupiter, and suggesting that textbooks in astronomy will have to be revised to show the total number of moons in the Solar System twenty-eight instead of twenty-six.

In addition to the most fascinating study of the beauties and grandeurs of the heavens, the scientific knowledge of the facts and principles of astronomy has been of great value in other fields of learning, and in the development and progress of modern civilization. The values of astronomy may be classified under the following five heads: practical values, scientific values, intellectual values, cultural values, and aesthetic values.

Among the practical values may be mentioned: 1. The safe navigation of the sea and air is absolutely dependent upon the applications of the facts and principles of astronomy; 2. The science of astronomy has enabled us to establish definitely the exact time units requisite for the calendar; 3. A knowledge of astronomy has made possible accurate calculations of the size and shape of the earth, thereby enabling us to make maps of the continents and the oceans; 4. International boundary lines in many cases are defined by latitude and longitude instead of by natural barriers; 5. The facts and principles of astronomy have assisted in the development of many other sciences, such as physics, chemistry, geology, higher mathematics, etc.; 6. The System of Standard Time was made possible by means of the facts and principles of astronomy; 7. The science of meteorology and the weather bureau, which assist in preventing millions of dollars of loss to the farmers, fruit growers, and stock raisers every year, are based upon scientific applications of astronomical facts and principles.

Some of the scientific values are: astronomy determines the methods to be used in solving problems in advanced mathematics. It enables us to compute the sizes, distances, movements, as well as the compositions, temperatures, etc., of other heavenly bodies. The past, present, and future development of the important field of geography has been and will continue to be dependent upon a knowledge of astronomical facts and principles.

It has been said that the greatest value of astronomy to the individual lies in its appeal to the intellect.[3] The contemplation of the great worlds outside of the earth; the study of the stars individually,

[3] Moulton, Forest Ray: *An Introduction to Astronomy*, The Macmillan Company, New York, N. Y., 1920; p. 18.

in clusters, or in vast systems; the realization that the universe is a universe of order subject to law; the gradual discovery of the laws in accordance with which it operates and the thoughts aroused as to its origin, its purpose, and its future development, will more than repay the investment of time and effort necessary to make them possible. In the first place, astronomy exalts truth and honestly seeks it wherever the search may lead. In the second place, its subject matter often gives a breadth of vision which is not obtained otherwise. Young says:

> The appreciation of the facts about astronomy cannot fail to possess an important influence in determining the attitude of the contemplative student towards such problems as man's obligations to future generations, his place in the universe, and *his relation to the Power behind it.*[4]

The study of astronomy has a cultural value. The increased knowledge of this subject has greatly eliminated superstitious beliefs in regard to meteors, comets, eclipses, and the planting of crops in particular phases of the moon. Notwithstanding the liberating effects on the minds of the intelligent and thinking individuals which the discoveries in the field of astronomy have brought, yet the great mass of the ignorant and unthinking may still become excited and panic stricken by the siren strains of Hill Billy music, or at some display of fantastic imagination of a publicity seeker in his attempt to picture the horrors of an Armageddon between the military forces of all the planets. A knowledge of astronomy broadens man's life, and gives him a proper sense of his place and relation on the earth and to the great universe about him.

The great French mathematician and astronomer, Laplace, in speaking of the beauties of astronomy said:

> Contemplated as one grand whole, astronomy is the most beautiful monument of the human mind, and the noblest record of its intelligence.[5]

And may I close with the words of one of the most eminent modern astronomers, Forest Ray Moulton, who said:

> The theory which gives unity to a great variety of observational data is of rare aesthetic value. It is related to the catalogue of imperfectly correlated facts upon which it is based as a finished and magnificent cathedral is to the unsightly heaps of stone, brick, and wood from which it was built.[6]

[4] Young, Charles A.: *A Textbook of General Astronomy*, Ginn and Company, Boston, Massachusetts, 1916.

[5] Thomas, Oswald: *Heaven and Earth*, W. W. Norton & Company, New York, N. Y., 1930.

[6] *Ibid.*

IF THE OBJECT of general education is to train the mind for intelligent action, logic cannot be missing from it. Correctness in thinking may be more directly and impressively taught through mathematics than in any other way. . . .

We have then for general education a course of study consisting of the greatest books of the Western world and the arts of the reading, writing, and speaking, together with mathematics, the best exemplar of the processes of human reason.—By ROBERT MAYNARD HUTCHINS in "What Is a General Education?" from *Harper's Magazine* of November, 1936.

The Teaching of "Flexible" Geometry

By Daniel B. Lloyd
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There are many interesting instruments commonly known today the principles of whose design and operation are based primarily on plane geometry. As a partial answer to the two-fold problem of motivating the subject and of increasing its practical value, probably a number of these could be studied profitably during the usual tenth-year course.

Numbered among these are (1) the pantograph, an instrument for enlarging or reducing drawings; (2) the plagiograph, a skew pantograph, which not only reproduces a drawing to desired scale, but also turns it through any angle; (3) the inversor, for reversing a drawing; (4) Peaucellier's cell, composed of seven links, which traces a geometrically straight line, without depending on a "straight-edge" ruler; (5) other miscellaneous linkage instruments for drawing straight lines or other kinds of lines; (6) Kempe's angle trisector, and other trisection compasses; (7) parallel rulers, for mechanically drawing a series of parallel lines; (8) proportional compasses; and (9) the diagonal scale.

These and other similar linkage instruments well illustrate such basic geometric properties as parallelism, collinearity of points, equality of lines and angles, similarity of figures, symmetry, and congruence. They well show upon what these properties depend and of what conditions they are independent; what relative motions affect them and what relative motions do not. They show "Geometry in Action"; they set the geometric elements into motion;

"She moves, she breathes, she seems to feel
The thrill of life along her keel."

To illustrate how any of the above mentioned geometric instruments could be studied profitably in class, let us consider in detail one of them, the pantograph.

The pantograph (or pantagraph) from the Greek, *panta*, all, and *graphen*, to write, is an instrument for making a reduced, and enlarged, or an exact copy of a drawing or map. In its commonest form it consists of four arms jointed together as shown in Fig. 1. The arms KB and KC

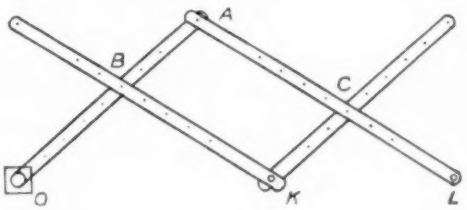

Fig. 1.

are arranged to be jointed with pins at a series of different positions to arms AO and AL, respectively, but AB must equal KC and AC equal KB so that $ACKB$ is a parallelogram whatever the angle at A. The pinholes on the arms are numbered in sets to indicate the proper positions for jointing the arms for enlargment of reduction ratios, such as $\frac{1}{2}$, $\frac{1}{3}$, which are commonly required.

The instrument is supported parallel to the paper on knob casters on which it moves freely. O is used as a fixed pivot about which the instrument turns and is arranged with an axle on a weighted fulcrum. The linkage is allowed to rotate in a horizontal plane about O. Then, as a stilus inserted vertically at K traverses a drawing, simultaneously a pencil fixed vertically at L will trace a similar figure on a larger scale. Or the pencil and stilus may be interchanged and the drawing reduced rather than enlarged.

In studying this instrument the interesting feature is the development of the proof in class to show that similar figures

will simultaneously be traced by the two pointers. In the proof which follows, it is assumed that the stilus is at the joint K; if instead it were anywhere along the arm BK the proof would vary but slightly. The only requirement is that L, O, and K, must be collinear.

It is desired to prove that in this linkage system K and L will simultaneously trace similar figures, L the larger, the ratio, of scales being $OA:OB$, due to every possible movement of the linkage in the plane.

Proof: **Part I. To show that O, K, and L, always remain in a straight line:**

If the linkage remains rigid and rotates about O, the straight line OKL is not affected (A figure may be moved from one position to another without affecting its size or shape). Therefore, consider only the rotation about A and B.

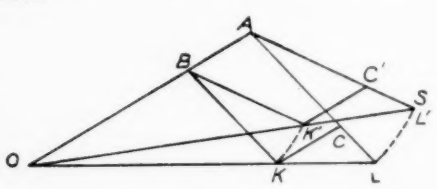

FIG. 2.

Rotate L to L' about A; this moves C and K to C' and K', respectively (see Fig. 2). To show O, K', and L' collinear:

$OB:OA::BK:AL$ (similar triangles) (1)
$OB:OA::BK':AL'$ (substitution) (2)

Product OK' to meet AL' at S; it will be shown that S coincides with L'. Since BK' is parallel to AS.

$$OB:OA::BK':AS \qquad (3)$$

Comparing steps (2) and (3), $AL' = AS$, and L' coincides with S. Therefore, $OK'L'$ is a straight line.

Part II. To show that the figures traced by K and L are always similar (see Fig. 3).

Let L move to any point L' by *any* combined rotation at O, A, and B. Then K, C, A, B, move to K', C', B', A'. $OK'L'$ remains a straight line (Part I, above).

$OB:OA::OK:OL$ (similar triangles) (1)
$OB':OA'::OK':OL'$ (similar triangles) (2)

but

$OB:OA::OB':OA'$ (identity) (3)

therefore
$OK:OL::OK':OL' = k$, say (Axiom 1) (4)
KK' is parallel to LL'

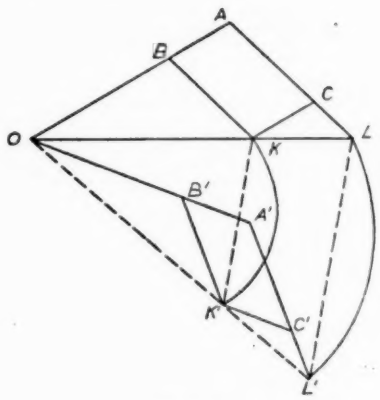

FIG. 3.

(since KK' cuts OL and OL' into prop. segments)
Thus, if KK' is a straight line, LL' is also; but if KK' is a broken or curved line, LL' must be likewise.

Triangles OKK' and OLL' are similar. (5)

Therefore

$$\frac{KK'}{LL'} = \frac{OK}{OL} = k \text{ (Cf. Step 4)}$$

Similarly, for any other positions of K and L, as K'' and L'', we have

$$\frac{KK''}{LL''} = k \qquad (6)$$

Therefore

$$\frac{KK'}{LL'} = \frac{KK''}{LL''} = \frac{KK'''}{LL'''} = k \qquad (7)$$

Any figure traced by K (or L) may be decomposed into a number of triangles of which KK' (LL'), KK'' (LL''), etc., are sides. Fig. 3 shows one such pair of sides, KK' and LL'.

For every such side traced by K there is a corresponding side traced by L. For the movements of K and L are linked and are interdependent. Then for each triangle in one figure there will be a corresponding triangle in the other.

Since the sides of these triangles are proportional (Step 7), the corresponding triangles are similar in pairs.

Therefore, the figures traced by K and L are similar (Two polygons are similar if they are composed of the same number of triangles, similar each to each, and similarly placed).

Part III. The ratio of the scales for the two fgures

$$= \frac{OB}{OA} \ (=k)$$

(Part II, Step 5)

The idea of studying geometric instruments may be extended, within limits, to an inductive approach to geometric principles, not replacing but merely supplementing the usual demonstrative method. Although the intuitive and inductive methods are supposed to have been treated in the earlier grades it is surprising how little has been absorbed. It would seem reasonable then to re-apply these methods in the tenth grade in a more mature way, as I have intimated. (For example, parallel rulers, improvised with elastic diagonals will suggest the theorems of the parallelogram.) A few simple instruments, either commercially made or designed in the school shops, will serve to *develop* as well as to illustrate the geometric principles and theorems.

In courses in trigonometry, instruments such as the surveyor's compass, the engineer's transit, and the sextant are commonly studied. Why cannot the students of plane geometry, many of whom never study trigonometry, be familiarized with a few of those instruments that are mainly geometric in principle? Though some of these ideas are not new, it is the author's belief that their more general practice might in a small way help to overcome the widespread fog of skepticism concerning the fundamental importance of geometry in the high school curriculum of today.

Traditionally, geometry has been studied in the tenth grade as a system of rigid, non-flexible elements. The author advances the thesis that *all* geometric diagrams should be assumed as *flexible linkages* and that, correlatively with the usual proofs, a study of the variance or invariance of properties of the diagram be made when any or all of the joints are flexed. To this pedagogic approach the author would attach the term "flexible geometry," and with the suggestion and hope that other educators may sense further possibilities for its development and use. It is not intended that the static diagram be eliminated in favor of the kinematic one, but the properties of the former can be elucidated by consideration of the latter, and incidental to the usual demonstrations.

Modern methods of teaching emphasize the placing of subject matter in a true life setting. This can not be as fully accomplished in geometry due to its abstractness. Whereas the truths of geometry remain through the centuries "The same yesterday, and today, and forever," it is the author's belief that they will be revealed more strikingly when developed by the above method.

A Study in Euclid

By ISADORE ELIZABETH FLANDERS
Central Junior High School, Saginaw, Michigan

A tree, bare, black against the sky,
Is also pleasing to the eye;
A study in geometry
In its revealed anatomy
Of prism-like trunk
Topped with circle, whose radii
Swing grace from arc to arc; where sly
Little angles, acute, obtuse
Play hide and seek, give no excuse
For such conduct!

"Pons asinorum" too, is there,
Light crossing it with dainty flair,
A flash of red-breast hunting keen
Four angles right to house his lean
Three hungry ones.

◆ THE ART OF TEACHING ◆

Mathematics Clubs

By H. VERNON PRICE
University High School, Iowa City, Iowa

UNDOUBTEDLY familiar to most teachers is the "popular" type of mathematics club wherein all pupils who so desire are welcomed as members. Due to the heterogeneity of talent which is embraced by such a plan, the group activities are in most cases limited to such things as mathematical recreations, puzzles, plays, and simple applications. The club has the advantage of appealing to large numbers, but also has the disadvantage of prohibiting discussions of an advanced nature.

Perhaps less familiar is the diametrically opposite type of club which limits membership but permits almost complete freedom of activity. A club of this type is now in operation in our high school and, although sponsored by faculty members, is student-controlled. The club constitution limits membership to a small number of carefully selected students. Admission requirements include both the passing of a comprehensive examination in mathematics constructed and administered by the sponsors and unanimous approval of the active members. Examinations are of such a nature as to exclude almost all students below the junior year. All members upon graduation become inactive but are entitled to attend meetings. Each member takes charge of at least one meeting per year at which time he gives a report on some mathematical topic. After the report the meeting is thrown open for discussion.

Even with the relatively homogeneous membershp which this club insures, great differences in interest are evident. These differences are reflected in the choice of such topics as: Men of Mathematics, Calendar Reforms, Mathematics in Clocks, Pi, Infinity, The Abacus, Number Systems, Methods of Angle Trisection, Fourth Dimension, Non-Euclidean Geometries, and Mathematics in Navigation. The amount of research done by some individuals is remarkable. Equally encouraging is the attitude of the student body. The question asked candidates for admission is not "Were you foolish enough to want to belong to the mathematics club?", but rather "Were you good enough to make the grade?"

The superior pupil is usually the most neglected pupil in our high schools and this exclusive type of club is organized for his benefit. It has been the purpose of this article to present the merits of such a club, at the same time recognizing that more popular types of clubs also have an important function in our schools. Any teacher who sponsors a club will be amply repaid for his efforts if he uncovers a future mathematician or if he helps many pupils to enjoy mathematics more fully.

National Council Members

ORDER your copy of the Fourteenth Yearbook on "The Training of Teachers of Mathematics" now. See back outside cover page of this issue for particulars.

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS CONVENTION PLANS

1. December 28 and 29, 1939—Columbus, Ohio

Joint meeting with The Mathematical Association of America and with The American Association for the Advancement of Science.
Theme: Relational Thinking in Secondary School Mathematics.
Headquarters: To be announced.
Program: Tentative given below, complete in next issue of THE MATHEMATICS TEACHER.

2. February 22 and 23, 1940—St. Louis, Missouri

Meeting with the American Association of School Administrators and its allied departments and organizations.
Theme: Mathematics for the "Other-than-College-Preparatory" Student.
Headquarters: Hotel Chase. (Get in your reservations at once.)
Program: Practically completed, a really *National* meeting, to be announced soon.

3. June-July 1940—Milwaukee, Wisconsin

Theme, headquarters and program not yet determined. Suggestions welcomed.
Suggestions and questions should be directed to E. A. Katra, Executive Secretary N.C.T.M., 525 West 125th Street, New York City.

THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Sixth Annual December Conference

Columbus, Ohio December 28 and 29, 1939

Theme: Relational Thinking in Secondary School Mathematics Teaching.
Headquarters: Hotel or dormitory to be announced.

Program in Outline

Thursday, Dec. 28, 6:30 P.M.

Joint Dinner Meeting with the Mathematical Association of America.
Speaker, place, and other details to be announced.

Friday, Dec. 29, 9:30 A.M.

A. Training Teachers for Relational Thinking
F. L. Wren, Chairman
B. Relational Thinking in Secondary Mathematics as Viewed by the College Teacher
Mary A. Potter, Chairman
Presiding: R. L. Morton
Speakers: F. L. Wren and A. A. Bennett
Other panel members: Charles Wiedeman, J. R. Overman, H. C. Christofferson

Friday, Dec. 29, 2:00 P.M.

A. Teaching Children to Do Relational Thinking
Presiding: Harold Fawcett
Speakers:
C. L. Thiele "In Junior High School Arithmetic"
Dale Wantling "In Algebra"
Marie Wilcox "In Geometry"
Other panel members:
Dorothy Wheeler, Hartford, Conn.
F. N. Harsh, Kent, Ohio
A. Brown Miller, Shaker Heights, Ohio
Martha Hildebrandt, Maywood, Ill.
B. Joint Meeting with the Mathematical Association
Speaker: Dr. Karl W. Bigelow, Chairman of the Commission on Teacher Education of the American Council on Education, 744 Jackson Place, Washington, D.C.
Panel members to be announced.

Note: These meetings will probably be held on the campus of Ohio State University. Room assignments have not yet been made to us by the American Association for the Advancement of Science. These we hope to announce in the next issue of THE MATHEMATICS TEACHER.

Program Committees:

A. Teacher Training—F. L. Wren, Nashville, and L. H. Whitcraft, Muncie
B. Secondary Mathematics—Mary A. Potter, Racine, Wis., Ralph Beatley, Cambridge, and Harold Fawcett, Columbus, Ohio

EDITORIAL

All teachers of mathematics and particularly those who have charge of the training of teachers of mathematics for the secondary schools should read the article by B. W. Jones of Cornell University, "On the Training of Teachers for Secondary Schools," which appeared in *The American Mathematical Monthly* for August-September, 1939, pages 428–434.

After referring briefly to "The Report on the Training of Teachers of Mathematics" which appeared in the *Monthly* (Vol. 42, 1935, pp. 263–277), Professor Jones goes on to set up "some principles which should underlie a program complete after one graduate year, a 'complete' program being one which completely prepares a student to continue education on his own hook."

A few salient points in the article will bear repetition here:

1. It is not my aim to quarrel with the two parts, as such, of graduate training: that is, first, the laying of a broad foundation of knowledge and method in the various fields of mathematics; and, second, specialized studies leading to and giving training in mathematical investigation. But any program for the training of teachers much be tied to what they are to teach. Breadth of knowledge and ability to do mathematical investigation are soon dissipated for the average teacher if the knowledge and the investigation are too far separated from what he teaches. One cannot effectively teach fractions in working hours and learn topology at other times; either fractions become dull or topology goes by the board. The path leading to research is, for most mathematicians, a *narrow* one. A teacher of mathematics in secondary schools needs *breadth* of outlook. To do research, one must be sophisticated—to teach, naïve. This is not to be taken to imply that teaching is incompatible with mathematical investigation. In fact, a teacher of any subject becomes dead as soon as he ceases to discover in his subject things new *to him*. But it does imply that research (as above defined) in mathematics is, for the most part, incompatible with good secondary school teaching. This is partly due to lack of time to engage in research, but the fundamental reason is the research is too far removed from the field of secondary school teaching.

2. Perhaps the most vital thing to any student of mathematics is insight. We are apt to take it for granted that piling course on course (regardless of the course) will accomplish this. We must realize that there are two parts to insight: first, understanding a thing with respect to itself; and second, understanding a thing with respect to its ramifications; and the first part must largely precede the second. If a student knows high school algebra only formally, he cannot obtain insight into the theory of equations, determinants, or the binomial theorem; nor, under these conditions, can the study of these more advanced subjects, however carefully taken, contribute greatly to an understanding of high school algebra.

3. Also the secondary school teacher needs a broader training than one who is to teach in college. As was mentioned in the report, he must know the role of mathematics in economics, statistics, and the sciences—things for which there is little place or time in the present graduate program. It is, in fact, this need, rather than the number of necessary courses in mathematics, which, with the required work in education and the training in mathematical investigation, make a fifth year of study increasingly necessary. While the student who is to go on to research can, in his undergraduate years, take elective mathematics courses, the prospective teacher must broaden the base of his knowledge.

4. We must realize that there are many ways to serve mathematics: advance its frontiers, recruit its army, spread its propaganda. They require different aptitudes, but who shall say that one is more important than the other! Contrast, for example, the general practitioner in medicine with one doing research in it. The profession is not well served if the medical scientist looks down on the practitioner or disparages his service any more than it is served if the practitioner proceeds without regard to the scientist. The latter depends upon the former for the application of his discoveries and, in many cases, for the testing of his results. The former knows many things the latter does not. To try to establish which is on a "higher level" is only to encourage division. Each must be self-respecting in his own sphere and have an appreciation of the vital contributions of those in other spheres. So it is, in a slightly different way, in mathematics. While a high school teacher seldom can contribute to the advancement of scientific knowledge, he has a delicate and vital service to mathematics to perform. To say that he teaches in high school because he isn't good enough to teach in college is similar to saying that he teaches because he isn't good enough to be a principal or superintendent. Administration and teaching require

different aptitudes, just as do college and high school teaching. It is a very difficult thing to give high school students a feeling for sound mathematics and develop ability in it. Later instruction is easy compared with it. (It is my private opinion that an undergraduate would learn much more college mathematics under a high school teacher who was competent in high school mathematics than would a high school student under a college teacher who knew well his college mathematics.) The choice is not between "higher" and "lower," but should be based on individual capabilities and inclinations. Only by co-operation (this implies meeting on a basis of equality) between college teachers and secondary school teachers can the best interests of mathematics be served and a completely worth-while program for teacher training be worked out.

It is doubtful that the majority of college teachers of mathematics will agree with the spirit of this article, yet it is very important right now that someone should have the courage and interest to say just what Professor Jones has said. Some prospective teachers of mathematics and teachers in other fields who have finished the work for the Ph.D. degree and who still are unable to find teaching positions cannot quite understand why they should be required to have more general qualifications. Experience in a teaching position will often convince one how much more is really needed than mere research ability or a highly specialized background of work.

This question of teacher training is not only one in which college and high school teachers should co-operate but also one in which all educators should be interested.

The National Council Yearbook for 1939 is devoted to teacher training in mathematics (see back outside cover of this issue of THE MATHEMATICS TEACHER) and, if generally distributed and read, will help greatly in getting before us in clearer form what the guiding principles in training teachers of mathematics should be.

W. D. R.

The National Council of Teachers of Mathematics

Official Notice

AS SECRETARY of the National Council of Teachers of Mathematics, I officially announce the annual election of certain officers of the National Council, said election to take place at St. Louis, Missouri, on Friday, February 23, 1940. Article III Section 7 of the by-laws states: "At least two months before the date of the annual meeting, all members shall be given the opportunity, through announcement in the official journal, to suggest by mail for the guidance of the directors a candidate for each elective office for the ensuing year. At least one month before the annual meeting the secretary of the board of directors shall send to each member an official ballot giving the names of two candidates for each office to be filled. These candidates shall be selected by a nominating committee of the board of which the secretary shall be chairman. The election shall be by mail or in person and shall close on the date of the annual meeting."

At the Atlantic City meeting, February 26, 1938, of the National Council the nominating committee consisting of the two most recent ex-presidents and the secretary as chairman (for this year J. O. Hassler, Martha Hildebrandt, and Edwin W. Schreiber), was instructed to prepare an official ballot suggesting two eligible candidates for each elective office and reserve a blank space for a third prospective candidate which may be written in by the voter. The officers to be elected at the St. Louis meeting are: President, 1940–41; Second Vice President, 1940–41; and three Directors, 1940–42. Since there is to be no primary ballot, affiliated organizations, local clubs, or individual members who wish to present names to appear on the official ballot should do so by writing to the Secretary immediately. The official ballot will be sent to members through the mail the first week in January.

The periods of service of the officers of the National Council, from its organization in 1920 to the present time, are printed below.

EDWIN W. SCHREIBER, *Secretary*

The National Council of Teachers of Mathematics

Organized 1920—Incorporated 1928

Periods of Service of the Officers of the National Council

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John P. Everett, Kalamazoo, Mich., 1930–1931
William Betz, Rochester, N. Y., 1932–1933
J. O. Hassler, Norman, Okla., 1934–1935
Martha Hildebrandt, Maywood, Ill., 1936–1937
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E. R. Breslich, Chicago, 1939–1940

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Elizabeth Dice, Dallas, Tex., 1928, 1929–1931
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John R. Clark, New York City, 1929–1931
Mary S. Sabin, Denver, Colo., 1929–1930, 1931–1933
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C. Louis Thiele, Detroit, Mich., 1931–1933
Mary Kelly, Wichita, Kan., 1932
John P. Everett, Kalamazoo, Mich., 1932–1934
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Raleigh Schorling, Ann Arbor, Mich., 1932–1934
W. S. Schlauch, New York City, 1933–1935
H. C. Christofferson, Oxford, Ohio, 1934–1936, 1937–1939
Edith Woolsey, Minneapolis, Minn., 1934–1936, 1937–1939
Martha Hildebrandt, Maywood, Ill., 1934–1935
M. L. Hartung, Madison, Wis., 1935–1937, 1938–1940
Mary A. Potter, Racine, Wis., 1935–1937
Rolland R. Smith, Springfield, Mass., 1935–1937, 1938–1940
E. R. Breslich, Chicago, Ill., 1936–1938
L. D. Haertter, Clayton, Mo., 1936–1938
Virgil S. Mallory, Montclair, N. J., 1936–1938, 1939–1941
Kate Bell, Spokane, Wash., 1938–1940
A. Brown Miller, Cleveland, Ohio, 1939–1941
Dorothy Wheeler, Hartford, Conn., 1939–1941

* Deceased

IN OTHER PERIODICALS

By NATHAN LAZAR
The Bronx High School of Science, New York City

1. Burton, E. D., "An Experiment with a Different Textbook." *School Science and Mathematics*, 39:529–532. June, 1939.

The writer reports the results obtained by teaching two classes in geometry—one used the regular textbook, and the other used Swenson's *Integrated Mathematics with Special Applications to Plane Geometry*.

A table is included which shows the I.Q. and the score made by each of the pupils of both classes on two tests. But the conclusion arrived at is that "the experiment was not extensive enough to draw any valid conclusions. My own notion is that it shows one can teach more or less what he tries to teach."

2. MacLane, Saunders, "Symbolic Logic." *The American Mathematical Monthly*, 46: 289–296. May, 1939.

This article is hidden away in the department "Mathematics Clubs" as a possible topic to be discussed at the meetings of clubs. It is one of the most lucid expositions of many difficult problems in symbolic logic that this reviewer has read.

The following topics are briefly discussed and references given for further study: paradoxes, calculus of propositions, propositional functions, formalization of mathematics, definitions of number, logic of real numbers, Boolean algebra, intuitionism, geometry and logic, consistency proofs, Gödel's theorem, and miscellaneous topics.

A bibliography of sixty items is included.

3. MacKay, David L., "The Lehmus-Steiner Theorem." *School Science and Mathematics*, 39: 561–572. June, 1939.

The Lehmus-Steiner theorem is none other than the well-known proposition that "If the bisectors of the angles at the base of a triangle, measured from the vertices to the opposite sides are equal, the triangle is isosceles."

The author not only gives a fascinating historical sketch of the origin of the theorem, but also points out the ramifications it has. Thus the phrase "bisectors of the angles at the base" is equally applicable to the exterior or the interior angles.

Steiner "considered the cases when the feet of the two external angle bisectors are (a) both above the base or both below the base, (b) on opposite sides of the base, and (c) when the external bisector from one end of the base equals the external bisector from the other end. He proved that for (a) the triangle is always isosceles, pointed out that for (b) it is not isosceles, and that for (c), although the triangle in general is not isosceles, it may be isosceles for triangles of a special species. Finally he generalized the theorem and discussed the theorem for the spherical triangle."

The proofs of the theorem are arranged in accordance with the following categories:

(a) Proofs based on the angle-bisector formula.
(b) Direct geometric proofs based on Book I.
(c) Indirect proofs based on Book I.
(d) Direct geometric proofs based on Book II.
(e) Indirect geometric proofs based on Book II.
(f) Direct proofs based on Book III.
(g) Indirect proofs based on Book III.
(h) Direct proofs based on Book IV.
(i) A more general form of the theorem.
(j) Trigonometric proofs.

Each of the proofs is given in complete detail together with the relevant historical and bibliographical references.

A bibliography of twenty-five items concludes the article. It is unfortunate that the author did not include the thirty odd references given by J. S. Mackay in the *Proceedings of the Edinburgh Mathematical Society*, 20: 18–22 (1901–1902).

It is to be hoped that the writer of the article will expand it into a more detailed monograph. It will surely receive as welcome a reception among teachers of geometry as this article will.

4. Rickey, F. A., "A Problem in Minimum Values." *National Mathematics Magazine*, 13: 362–366. May, 1939.

A discussion of the solution usually offered in textbooks of the following typical problem: "A man is in a boat 5 miles from a straight shore line. He can row at the rate of 3 miles per hour and walk at the rate of 4 miles per hour. How

far from the point on the shore directly opposite him should he land in order to reach a point s miles down the shore in the shortest time?"

The defects of the customary analysis are pointed out and an alternative one is presented.

5. Shaw, Allen A., "An Overlooked Numeral System of Antiquity." *National Mathematics Magazine*, 13: 368–372. May, 1939.

The purpose of the paper is to give a concise account of the Armenian numeral system which seems to have been overlooked by historians of mathematics. The Armenian numeral system consisted in assigning the first nine letters of the alphabet to units, the second nine to tens, the third nine to hundreds, and the last nine to thousands. Although the system did not have a zero, it did have local value. The Armenian alphabet system, the writer believes, is more perfect than any of the other ancient alphabetic systems. The only missing symbol is zero which was discovered and used later in the eighth century. He holds, moreover, that the origin of our common numeral system is *Armenian* or *Graeco-Armenian*, worked out at the court of the Baghdad Califs, possibly under the patronage of Harun-al-Raschid (c. 786 A.D.).

Another paper on this topic is promised.

6. Solomon, Charles, "Approximate Numbers." *School Science and Mathematics*, 39: 573–576. June, 1939.

After pointing out that there is a crying need for the competent teaching of approximate numbers, Solomon gives examples of unforgivable errors that are constantly being committed by engineers who should know better. He then quotes the material that is given to each of the teachers of a Brooklyn high school to "encourage" them to teach this topic in their algebra classes.

It is to be regretted that there is so much disagreement among the writers on approximate computation and approximate numbers that the conscientious teacher is frightened when he considers incorporating any of the material in his class work.

7. Stewart, Minnie, "Mathematics in the Progressive Education Workshop at Denver." *Bulletin of the Kansas Association of Teachers of Mathematics*, vol. 13, no. 3, February, 1939, pp. 8–9.

An enthusiastic description of the method of work and of the accomplishments possible at a summer workshop.

8. Vaughan, Adah, "Professor Whiz and His Class in Math Magic." *School Science and Mathematics*, 39: 540–545. June, 1939.

A little skit which may be added to any mathematics program given in an assembly or at a meeting of the mathematics club. The dialogue is all in rhyme, and contains many familiar and unfamiliar riddles and puzzles, suitable for a junior high school audience.

9. Wasser, Sara Belle, "An Experiment in Field Mathematics." *Bulletin of the Kansas Association of Teachers of Mathematics*, vol. 13, no. 2, February, 1939, pp. 7–8.

An interesting description of an elective given to a group of students who completed the ninth year algebra course. The author describes the equipment used, the method of group organization, the units worked out, and the benefits derived from the course.

Central Association Meeting!

THE ANNUAL meeting of the Central Association of Science and Mathematics Teachers will be held at the Morrison Hotel, Chicago, Illinois, on November 24 and 25, 1939. The October number of *School Science and Mathematics* contains a rather detailed description of the program. Eminent speakers in physics and chemistry will be included.

Teachers of Mathematics will be interested in the program of the Mathematics Section. Dr. Maurice S. Hartung will speak on "Issues Raised by Recent Reports on the Mathematics Curriculum." Dr. E. R. Hedrick, Vice-president and Provost of the University of California at Los Angeles, is scheduled to speak on the general program and on the Mathematics Section program, as is also Dr. Raleigh Schorling, Head of the Department of Mathematics in the University High School, University of Michigan.

Speakers for the Biology, General Science, and Elementary Science Sections were published in the June number of *School Science and Mathematics*. A more complete program will appear in the November issue of *School Science and Mathematics*.

NEWS NOTES

The 1939 American Education Week observance will be held November 6–11, 1939. "Education for the American Way of Life" is the general theme. As in previous years the National Education Association has prepared materials to assist schools in planning for this observance including colorful posters, leaflets, stickers, and packets containing special folders for the different school levels prepared by field committees in various sections of the United States, useful alike to the classroom teacher, principal, superintendent, or American Education Week committee. Early planning will help to make your observance most effective. For complete information, write to the National Education Association, 1201 Sixteenth Street, N. W., Washington, D. C.

The New York Times of September 29 carries an account of mathematics curriculum changes in the junior high schools of New York. Excerpts from the article follow.

"Changes in the curriculums of eighty-two junior high schools in the city in an attempt to bring more closely together the academic and commercial courses of study and to alter the study of mathematics by placing greater emphasis on the study of geometry are being considered by officials of the Board of Education.

"The attention of superintendents, directors, principals and heads of bureaus has been called to a proposed course of study in mathematics, approved by the Board of Superintendents, for experimental use in the junior high schools, by Dr. Harold G. Campbell, Superintendent of Schools. Dr. Campbell pointed out that the proposed course of study was sent to teachers of mathematics so that they might acquaint themselves with the content 'in anticipation of the adoption of the course next term.'

"The new course emphasizes the study of geometry and the approach to the study of algebra for the Grades 7A to 9B. The course is designed to develop ability to interpret and master the simple mathematical situations met by the average person in his home, business and community.

"It also attempts to utilize and extend the knowledge acquired in the preceding grades, to understand the importance of graphic representation in everyday life, and to discover the presence of and become acquainted with geometric forms common in nature, industry and everyday life."

The American Mathematical Society is in the fortunate position of having the financial backing, through the munificence of two of the great Foundations, to found a new international mathematical abstracting journal to be known as *Mathematical Reviews*. During the past quarter-century while the United States and Canada have been gradually assuming a more prominent part in mathematical research, there has been sentiment expressed from time to time among mathematicians that there should be a review journal sponsored by American organizations. But doubts as to whether the scientific and financial resources could be spared caused the postponement of the undertaking. However, the rapid growth of American mathematical resources and the availability of funds have resolved these doubts, and it has been decided to proceed immediately.

The first number of *Mathematical Reviews* is to appear late in 1939 or early in 1940; the material to be reviewed begins with the latter half of 1939. It is proposed to review all fields of pure mathematics and also those of applied mathematics and mathematical physics which are of pronounced interest to mathematicians. The new journal, which will be issued approximately once a month, will contain several thousand reviews annually and will run to approximately eight hundred large double-column pages. Professors J. D. Tamarkin and Otto Neugebauer will be the first editors. A strong group of collaborators for the initial period is assured.

The Carnegie Corporation has appropriated $60,000 as a backlog for the new journal. The Rockefeller Foundation has made a gift of $12,000 to cover some of the initial costs. Brown University is housing the project and aiding in the editorial work. The American Mathematical Society and the Mathematical Association of America are each starting off with a subsidy of $1,000 for the first year. Annual subsidies are being sought from other organizations, and plans for the permanent financing of the project are being considered. On account of the generous subventions, the subscription price will be set drastically below actual cost.

Partly with a view to aiding indirectly in the support of this journal, the Rockefeller Foundation has made a handsome gift to Brown University for an experiment in the dissemination of mathematical publications through the distribution of microfilm. This money is to be used

to augment the mathematical library at that university, a collection which is already internationally known as outstanding. Out-of-print journals will be put on film and made available to mathematicians; rare books of general use will be filmed; on request from a subscriber to the new journal, any article reviewed will be sent on film or as film-print. This service will be extended to all parts of the world at a price not exceeding cost. It should be of greatest value to mathematicians located in the smaller universities and colleges and should be a factor in encouraging young men to continue with their investigations. This interesting experiment in the promotion of a new aid to learning should prove to be an asset not only to *Mathematical Reviews*, but also to American mathematics in general.

The annual meeting of the Buffalo section of the National Council of Teachers of Mathematics was held April 26, 1939, at the Buffalo Athletic Club.

Aaron Buchman, Hutchinson Central High School, presided. The roll call expected a one-minute speech from each person. Miss Agnes Creahan and Miss Natalie L. Round gave five-minute reports on a meeting of the Progressive Association to which they went as delegates.

The Report of the Joint Committee appointed by the Mathematical Association of America and the National Council of Teachers of Mathematics was reviewed in sections by Miss Alberta Wanenmacher, Hutchinson Central High School; Miss Mary Kenny, East; Miss Sarah Coonley, Grover Cleveland; Miss Harriet E. Bull, Fosdick-Masten Park; Miss Ernestine McDonald, Lafayette; Miss Christiania Hathaway, Niagara Falls; Miss Sara Walsh, East; and Mr. Louis Scholl, South Park.

Mary E. Crofts

The Colorado Branch of the National Council of Teachers of Mathematics held its annual Spring meeting in Denver with an attendance of one hundred at the luncheon. Dr. L. R. Gustaffson of Colorado University was the speaker. Dr. Gustaffson, himself a scientist, urged a more thorough foundation in mathematics for students entering fields other than engineering. Too often, according to Dr. Gustaffson, we think that mathematics is of use only to those entering a highly specialized field; when in reality mathematics is indispensable not only in science and chemistry, but in biology, medicine and allied subjects whose very foundation is based upon mathematics. Those fields today become fields of dynamic inquiry; therefore they become mathematical. Teachers would do well to face the fact that today as never before, mathematics is essential to many fields. Teachers should encourage rather than discourage mathematics for the students. The science, the biology, and the medicine of today are not that of twenty years ago dealing in theories and generalities; but instead are increasingly demanding mathematical accuracy and scientific thinking.

According to Dr. Gustaffson, medical students without a good mathematical foundation before entering college are totally unfit and unprepared to pursue a serious study of modern medicine. He suggests the need of an elimination test to weed out those unprepared mathematically before they attempt the study of medicine.

Robert Thibodeau of Horace Mann Junior High and Miss Stancliffe of Byers Junior High reviewed the high spots of the Joint Report. Dr. C. M. Hutchinson a member of the committee led the discussion, and invited suggestions and criticisms concerning the work of the committee.

Mr. Charlesworth of East High School, Denver, introduced two members of the Euclideans Club of that school. These two boys presented an interesting and convincing discussion using charts and illustrations to show the advantages of the duo-decimal system. Several exhibits from the East High Euclidean Club exhibit were on display at the meeting. An exhibit of textbooks and mathematical literature was also shown.

The Colorado branch has two meetings during the year, one in the fall, the other in the spring. Meantime, discussion clubs and informal groups meet for discussions on subjects pertaining to mathematics, more effective teaching, and meeting the mathematical needs of society in a scientific age. During the school year a mathematical bulletin is published by the Mathematics Section of the Colorado Education Association.

The Euclideans Club of East Denver, Colorado, High School under the sponsorship of H. W. Charlesworth held its first annual mathematics exhibit in the spring of 1938. The success of the venture was such that the exhibit remained on display for three days. The enthusiasm shown by teachers and students as well as from the public has convinced the Euclideans that what started as an experiment will undoubtedly continue as an annual spring mathematical exhibit.

"I never realized there could be so many interesting exhibits shown in the field of mathematics" was a comment frequently heard. An evening meeting featuring the beloved Father Forstall, eighty-two year old scientist and mathematician of Regis college, in a discussion

"Mathematical Curiosities" held the audience spellbound for more than an hour.

East High students gave lectures in various class rooms on such subjects as "Mathematics in telepathy" "Trisection of an angle" "Duodecimal system" and "Calendar reform."

Sound pictures of a mathematical and scientific nature were shown throughout the exhibit. New and different items were added from day to day. Mathematics in engineering was illustrated by minature dams loaned by the Reclamation Bureau. Commercial companies, downtown stores, engineering companies, private collections, the museums, all co-operated generously in loaning unusual displays for the exhibit. Since the Denver Schools prohibit advertising at the schools, co-operation of the business houses and commercial companies was most generous. Valuable and rare exhibits were loaned by Mr. C. S. Ionides, consulting engineer, who has made a hobby of seeking out the rare and unusual. Translations made by Mr. Ionides from rare old works were displayed. Actual meteorites from the city museum were shown with a model demonstration for computing the height of a meteorite. The use of the multiple circle arch in engineering was demonstrated by model dams loaned by engineering companies.

"The odds against you in games of chance" provided a colorful and most attractive display supplemented by mathematical proof graphically presented. Here, presented in picturesque form was "enough material for a doctor's thesis or a dozen sermons" as one visitor commented. Artistic posters and beautifully colored designs added much to the attractiveness of the whole display. A play of colored lights on especially attractive and outstanding exhibits gave an impression of planning, vision and a complete picture in the minds of those planning the exhibit as a whole. From an artistic view, as well as the scientific, the showing was highly successful. No effort was spared in arranging the features artistically.

Students took charge of demonstrating the working models and explaining the features of the exhibit. The evolution of the calendar, calendar reform, evolution of the number system are only just a few of the many interesting features shown. A young Chinese dentist contributed a Chinese abacus such as was used in the years B.C. Students were instructed by Dr. Ling in the use of the instrument. A group of boys spent several evenings being instructed by a manufacturing company in the use of the calculating machines displayed by the company. The success of the exhibit was unquestionable. Even those who would be inclined to scorn such a pioneering enterprise paused to marvel at the worthwhileness of the project. Plans to make it an annual affair are under way.

It was by no means an exhibit of students' class work; but rather a practical application and demonstration of mathematics in actual life giving a suggestion of the romance of the science and its actual application to civilized man.

The report has been received of a Committee of the Mathematics Section of the Association of Private School Teachers appointed to work out a plan for facilitating the transfer of students who go from one school to another while preparing for college and the new requirements in mathematics for college entrance. The report follows in full.

An Attempt to Reduce the Difficulties of Pupil Transfer

To deal with these well known difficulties in their relation to mathematics, the following committee was appointed in December, 1937, by the Chairman of the Mathematics Section of the Association of Private School Teachers: Chairman, Lucy K. Roberts, The Masters School, Dobbs Ferry, New York; Elizabeth Cooper, Hunter High School, New York; Helen Crossman, Packer Collegiate Institute, Brooklyn, N. Y.; Irman H. Kaufmann, Bentley School, New York; Sarah Y. Keyser, Nightingale-Bamford School, New York.

The committee constructed a report consisting of a statement of aims and minimum essentials for a mathematical program in Junior and Senior high schools and caused it to be sent for suggestions and criticism to 80 principals of private schools; to Heads of the Mathematics Departments of Barnard, Bryn Mawr, Smith, Vassar and Wellesley; to Professor George W. Mullins, Secretary of the College Entrance Examination Board; to Professor Ralph Beatley, Chief Examiner for the Mathematics Attainment Tests; to Doctor Arthur Roberts, Chairman of the Secondary Education Board; and to the mathematics teachers in the Association of Private School Teachers.

All but one of the responses, more than 80 in number, were very encouraging. For example, Professor Mullins wrote:

"Your committee has, I am confident, performed a great service to the schools and to the colleges as well, in presenting such a clear and concise statement in regard to the philosophical outlook, the intellectual goal, and the purpose of the study of mathematics. These statements are excellent in every way.... All college teachers of mathematics should be and are, I believe, greatly interested in your problem, which after all, is our problem as well. There can be no ques-

tion about the importance of certain minimum essentials and these I believe, you have stated admirably. These essentials should be regarded as a common core about which the teacher works. I think I have no additions, to suggest. The great problem for the teacher, both in school and in college, is to teach the subject of mathematics as you have so well indicated, not as an end in itself but as a contributing factor to all those influences that enter into the growth of the pupil."

Professor Beatley has written:

" . . . I think it (the report) distinctly good. Your committee does well not to suggest the order of topics within a grade. . . . I know that you understand that in heartily approving this syllabus I speak as one who is principally concerned with the training of teachers of mathematics in secondary schools. Speaking as an agent of the College Entrance Examination Board, I must limit myself to the colorless but accurate statement that the syllabus is adequate. I will trust you to make clear to the rest of your group that although I think this syllabus is very fine indeed, I should be sorry if every school in the country should follow it; though I hope that *most schools* will do so; for, speaking both as Dr. Jekyll and Mr. Hyde, I am interested to see the avenues of progress kept open by significant differences and experiments in the schools."

And on the margin of the report Professor Beatley has penciled:

"I have studied 10th and 11th grades carefully—indeed all the grades—and I think it is *excellent*. I like it."

From Doctor Roberts of the Secondary Education Board comes the following:

" . . . What does impress me is that we have in it (the report) an attempt at solution of a pressing problem. If the values of continuity in instruction are to be realized, there must be some greatest common denominator of purpose and practice or pupils will suffer needless hardship when they pass from one school to another. You may be sure that the Secondary Education Board will co-operate fully with the other associations attacking a present obstacle to effective instruction and proper transfers."

Some of the responses offered suggestions which were embodied in the revised report as given below.

As approvingly noted by Professor Beatley, the Committee offers no recommendations as to the order of teaching the subjects listed for the various grade levels, or the time to be devoted to each, or the methods to be used, or the additional work which teachers will choose to include in their courses. It will welcome suggestions, criticisms and inquiries. Any questions teachers may wish to submit to the Committee will receive prompt answers, if a stamped addressed envelope is enclosed. Such letters or requests for additional copies of the outline may be addressed to Lucy K. Roberts, The Masters School, Dobbs Ferry, New York.

The Mathematics Section of the Association of Private School Teachers met on December 8, 1938, at the Bentley School, to hear the report of the Committee on Minimum Essentials for a Mathematics Program in Junior and Senior High Schools. The meeting was attended by representatives from 30 private schools located in New York, New Jersey, Connecticut, Pennsylvania and Virginia. The following resolution was adopted.

"Whereas, the lack of uniform placement of mathematical subject-matter in the various schools renders difficult the transfer of pupils from school to school; and

Whereas, Education for co-operation is an important obligation of our profession; therefore,

Resolved, That, in so far as it may be practicable to do so, we will, for at least two years, include in our respective curricula the program presented in the report on Minimum Essentials for a Mathematical Program in Junior and Senior High Schools."

Report on Minimum Essentials for a Mathematical Program in Junior and Senior High Schools

Foreword

Realizing keenly the inadequacy of any syllabus as a guide in teaching secondary mathematics to pupils who may be preparing for the Alpha, Beta or Gamma College Board Examinations, the committee feels constrained to suggest the following philosophical outlook for the teacher, and for the pupil an intellectual goal.

Philosophical Outlook

Mathematics is a language which has an old literature rich in beauty and content. This literature embodies what is, possibly, the most austere and beautiful of man's intellectual achievements. It is old, and it is yet young, for its content has been, and is still, growing with ever increasing acceleration. Merely to speak or write or read the language of mathematics, the only language by which much of the world's knowledge can be expressed, and to gain a working knowledge and understanding of its content, it is necessary to think. A qualified teacher of mathematics must appreciate the fact that the subject is unsurpassed for its utility, for its value as an intellectual discipline, and for its appeal to the esthetic sense, making possible the physical structures of our civilization, exposing

and exemplifying the standard of logical rigor, and contributing essentially to the discovery and depiction of the wonders of the universe.

Intellectual Goal

No age is too young to begin forming habits. If one would win a place among the leaders of one's generation, one must early form the habit of thinking—thinking, not merely at the suggestion of someone else, but at one's own suggestion. One must initiate thought. There is no more inviting field for initiating thought than the field of mathematics. Here one may cultivate orderliness in things and in thoughts; accuracy, persistence and patience in ferreting out meanings and in following arguments; criticalness in the examination both of one's own ideas and those of others; and clarity of exposition. Finally, the student of mathematics may throw off the yoke of concrete subject-matter and explore the field of pure Relations, not only those which constitute the ultimate structure of the actual world but those constituting the structure of any possible world.

Purpose

The purpose dominating the continuity of this program is the growth of independence of thought on the part of the pupil, faith in his own self-help, little dependence on the teacher. To facilitate the transfer of students, it is suggested that any curriculum built up from the following minimum basis might be the expectation for the six years prior to college entrance.

7th Grade

Arithmetic
- Development of Fraction-Decimal concept
- Percentage
- Graph

Intuitive Geometry
- Experimental—with ruler, compass, protractor, etc.

8th Grade

Algebra
- Relationships as expressed by table, formula, graph and words.
- Extension of 4 fundamental operations to Algebra including fractions with monomial numerator and denominator.
- Application of 4 axioms to linear and simultaneous equations.
- Simple verbal problems.

Arithmetic
- Extension of Fraction-Decimal concept.
- Percentage (with verbal problems within the child's experience).
- Social Arithmetic (with verbal problems within the child's experience).
- Emphasis on commonsense checking.
- Graphs—construction and interpretation.
- Square root with reference to the Pythagorean Theorem.

Intuitive Geometry
- Extension of 7th Grade work.
- Three Dimensional Geometry.

9th Grade

Algebra
- Through factorable quadratics with emphasis on solution of verbal problems within the pupil's range without dependence on the teacher.
- Numerical Trigonometry of the right triangle without interpolation.

Arithmetic
- Extension of Percentage.
- Square Root by Factors.

Informal Geometry with special emphasis on formulae and nature of proof.

10th Grade

Algebra
- Algebra through Quadratics.

Arithmetic
- Problems and computations.

Geometry
- Classes taking Beta Examination at end of 11th year, Books I and II.
- Classes taking Beta Examination at end of 12th year, Book I with emphasis on originals.

11th Grade

Algebra
- Classes taking Beta Examination at end of 11th year, Logarithms, Series, Binomial Theorem, Conic Sections, etc.
- Classes taking Beta Examination at end of 12th year, Algebra necessary in solution of geometric originals.

Geometry
- Classes taking Beta Examinations at end of 11th year, Books III, IV, V.
- Classes taking Beta Examinations at end of 12th year, Books II, III, IV.

Respectfully submitted,

Lucy K. Roberts, *Chairman*
Elizabeth Cooper
Helen Crossman
Irma H. Kaufmann
Sarah Y. Keyser

Join the National Council of Teachers of Mathematics

I. The National Council of Teachers of Mathematics carries on its work through two publications.

1. *The Mathematics Teacher.* Published monthly *except in June, July, August and September.* It is the only magazine in America dealing exclusively with the teaching of mathematics in elementary and secondary schools. Membership (for $2) entitles one to receive the magazine free.

2. *The National Council Yearbooks.* The first Yearbook on "A General Survey of Progress, in the Last Twenty-five Years" and the second on "Curriculum Problems in Teaching Mathematics" are out of print. The third on "Selected Topics in Teaching Mathematics," the fourth on "Significant Changes and Trends in the Teaching of Mathematics Throughout the World Since 1910," the fifth on "The Teaching of Geometry," the sixth on "Mathematics in Modern Life," the seventh on "The Teaching of Algebra," the eighth on "The Teaching of Mathematics in Secondary Schools," the ninth on "Relational and Functional Thinking in Mathematics," the tenth on "The Teaching of Arithmetic," the eleventh on "The Place of Mathematics in Modern Education," the twelfth on "Approximate Computation," the thirteenth on "The Nature of Proof," and the fourteenth on "The Training of Mathematics Teachers for Secondary Schools"—each may be obtained for $1.75 (bound volumes), from the Bureau of Publications, Teachers College, 525 West 120th Street, New York City. All of the yearbooks except the first two (3 to 14 inclusive) may be had for $16.50 postpaid.

II. The Editorial Committee of the above publications is W. D. Reeve of Teachers College, Columbia University, New York, Editor-in-Chief; Dr. Vera Sanford, of the State Normal School, Oneonta, N.Y.; and W. S. Schlauch of Hasbrouck Heights, N.J.

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